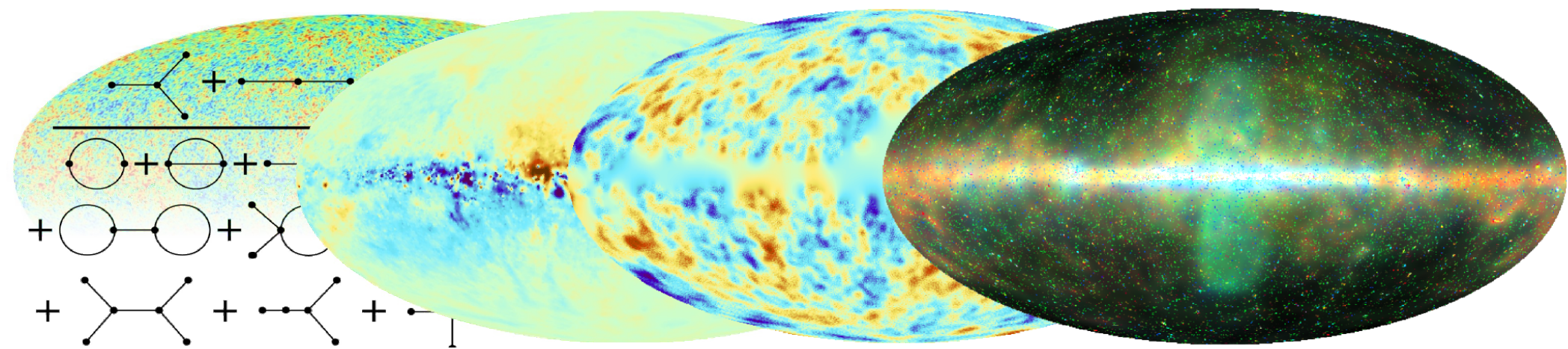
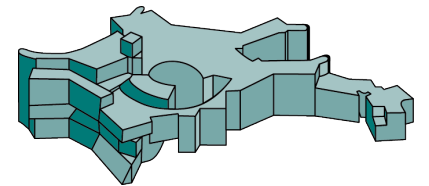




Information field theory



Torsten Enßlin
MPI for Astrophysics
Ludwig Maximilian University Munich

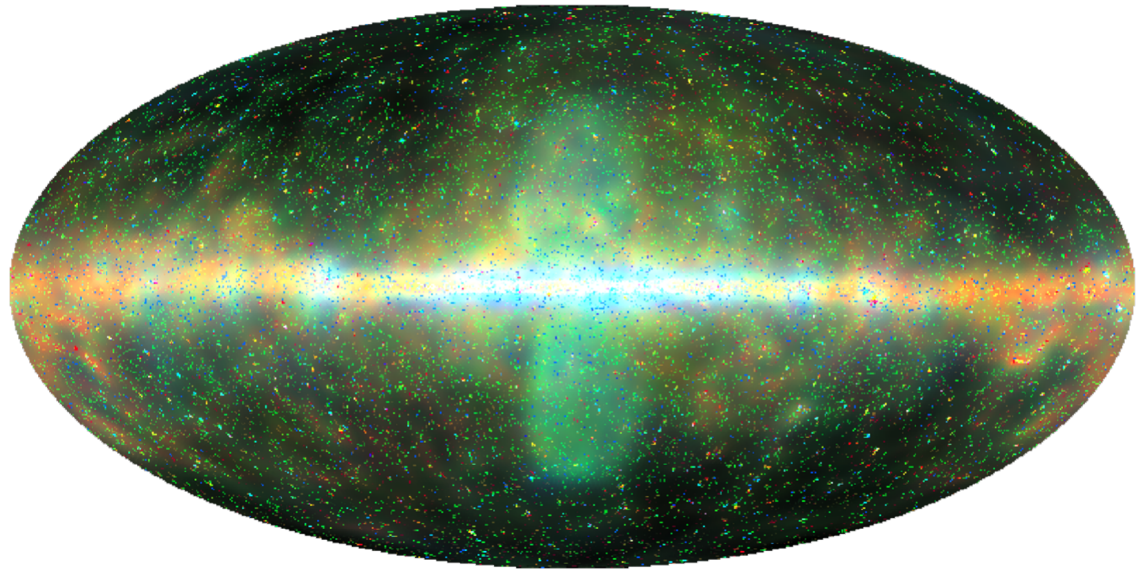
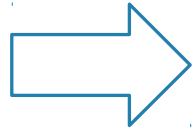


Photon count imaging

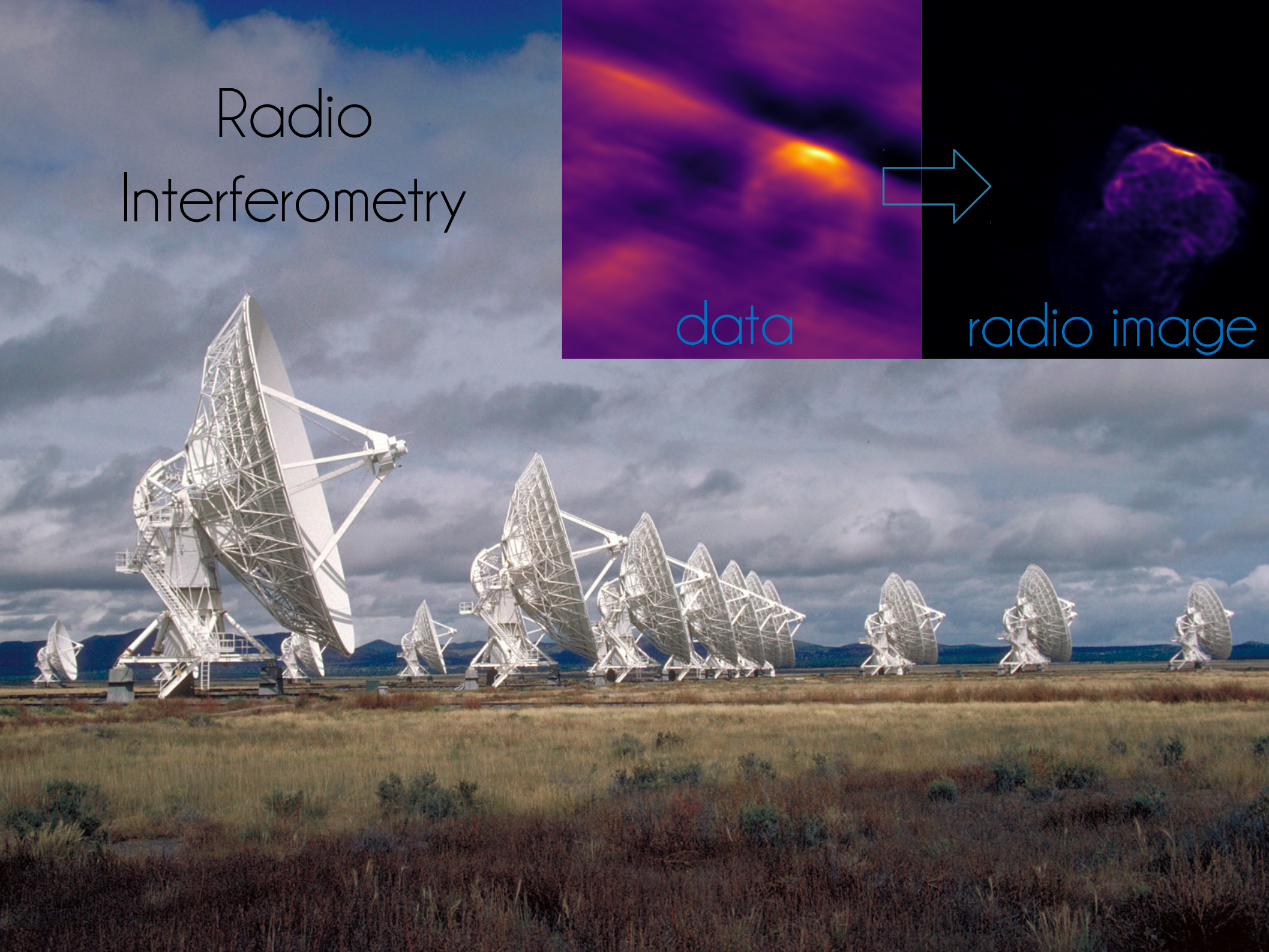
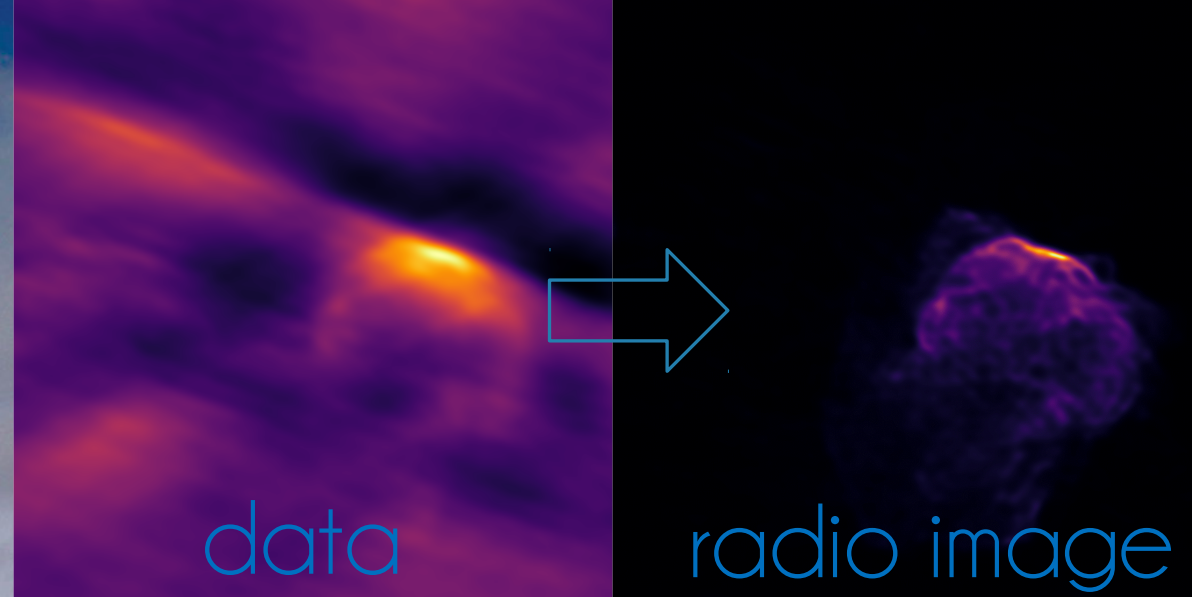
D³PO (Selig et al.)

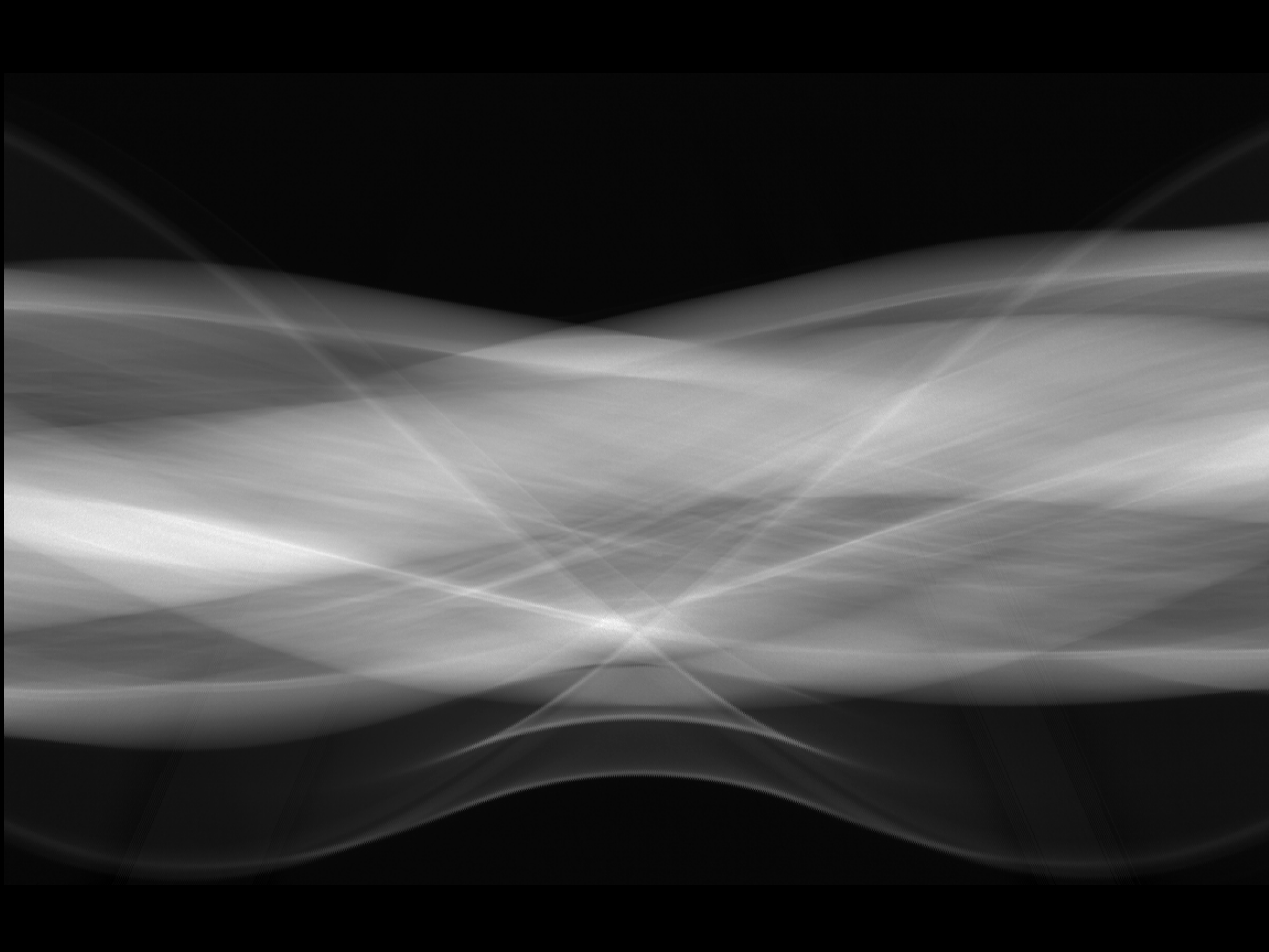


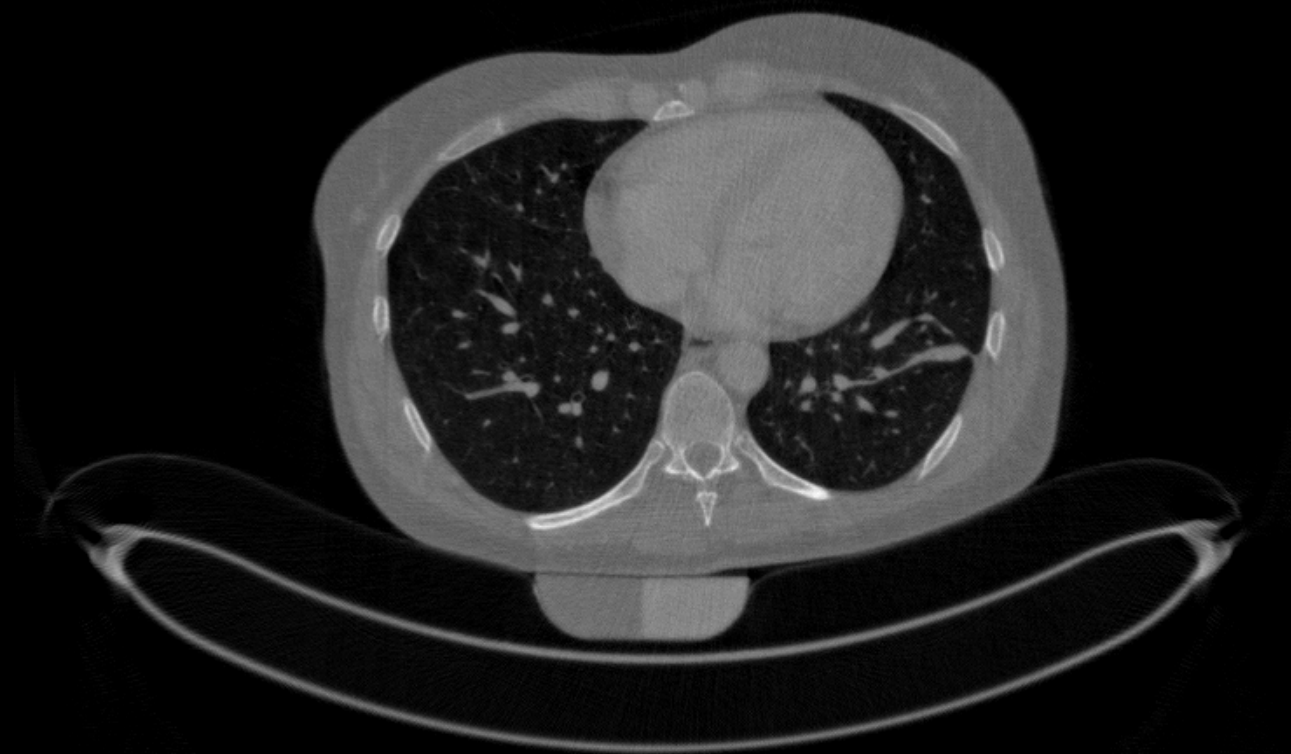
[Fermi - NASA/Kim Shiflett]



Radio Interferometry

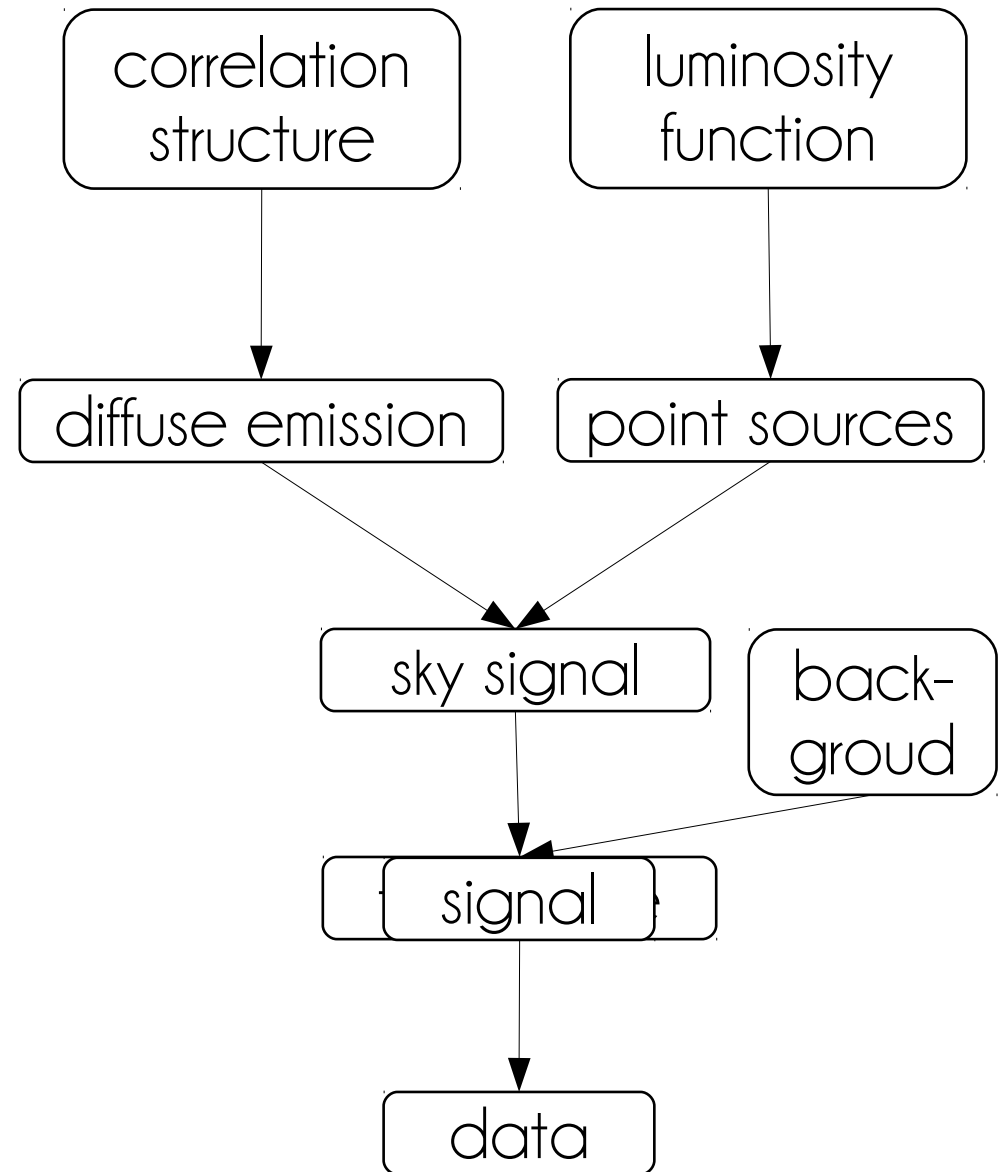


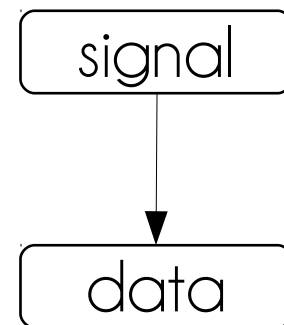
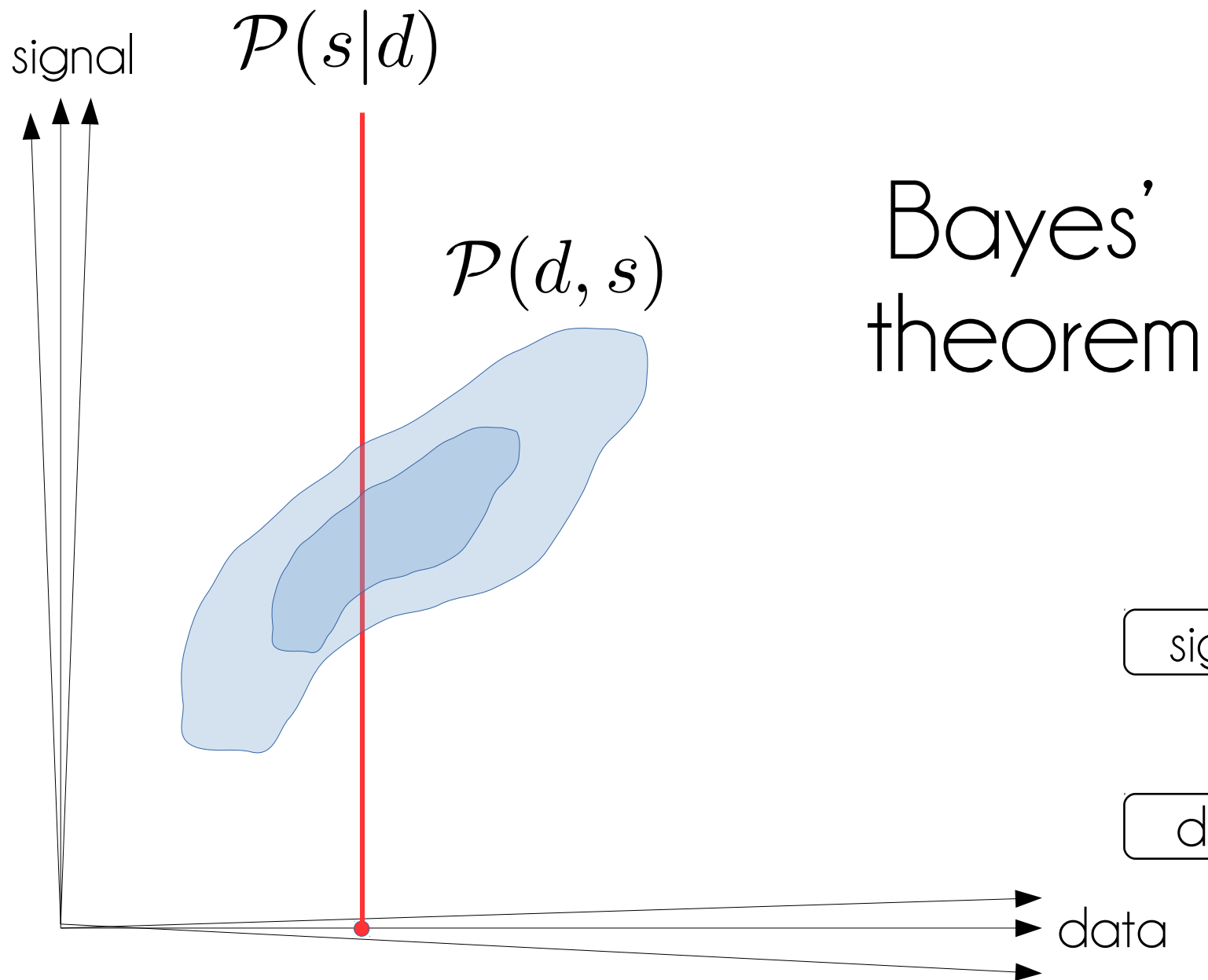






Measurement model





Information theory

$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d, s)}{\mathcal{P}(d)}$$

$$\mathcal{H}(d, s) = -\log \mathcal{P}(d, s) \quad \text{Information}$$

$$\mathcal{Z}(d) = \mathcal{P}(d)$$

$$= \int \mathcal{D}s \mathcal{P}(d, s)$$

$$\mathcal{P}(d, s) = \mathcal{P}(d|s) \mathcal{P}(s)$$

$$\mathcal{H}(d, s) = \mathcal{H}(d|s) + \mathcal{H}(s) \quad \text{is additive}$$

Probability & Information

$$\mathcal{P}(s) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{s^2}{2\sigma^2}}$$

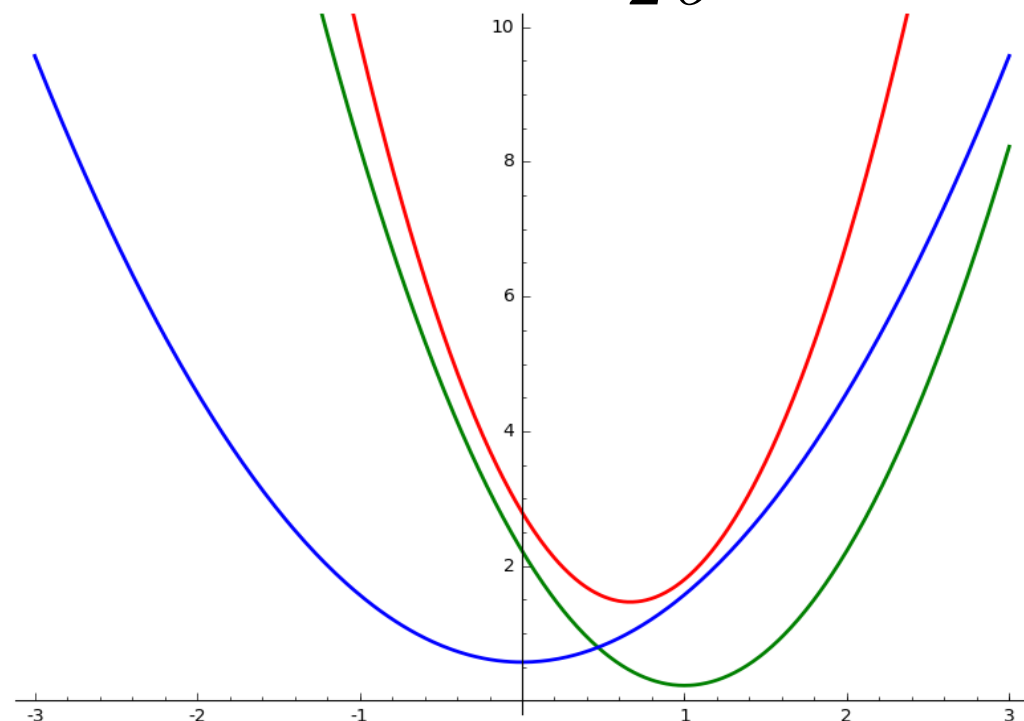
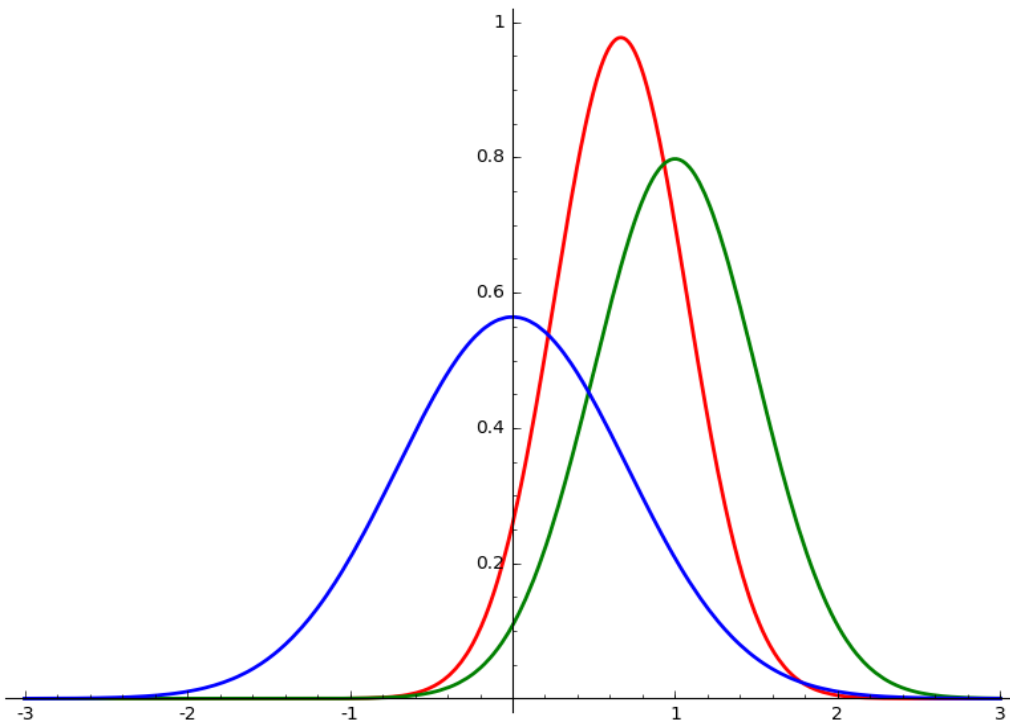
$$\mathcal{H}(s) \triangleq \frac{s^2}{2\sigma^2}$$

$$\mathcal{P}(d|s) \propto e^{-\frac{(s-d)^2}{2\sigma'^2}}$$

$$\mathcal{H}(d|s) \triangleq \frac{(s-d)^2}{2\sigma'^2} \sigma^2$$

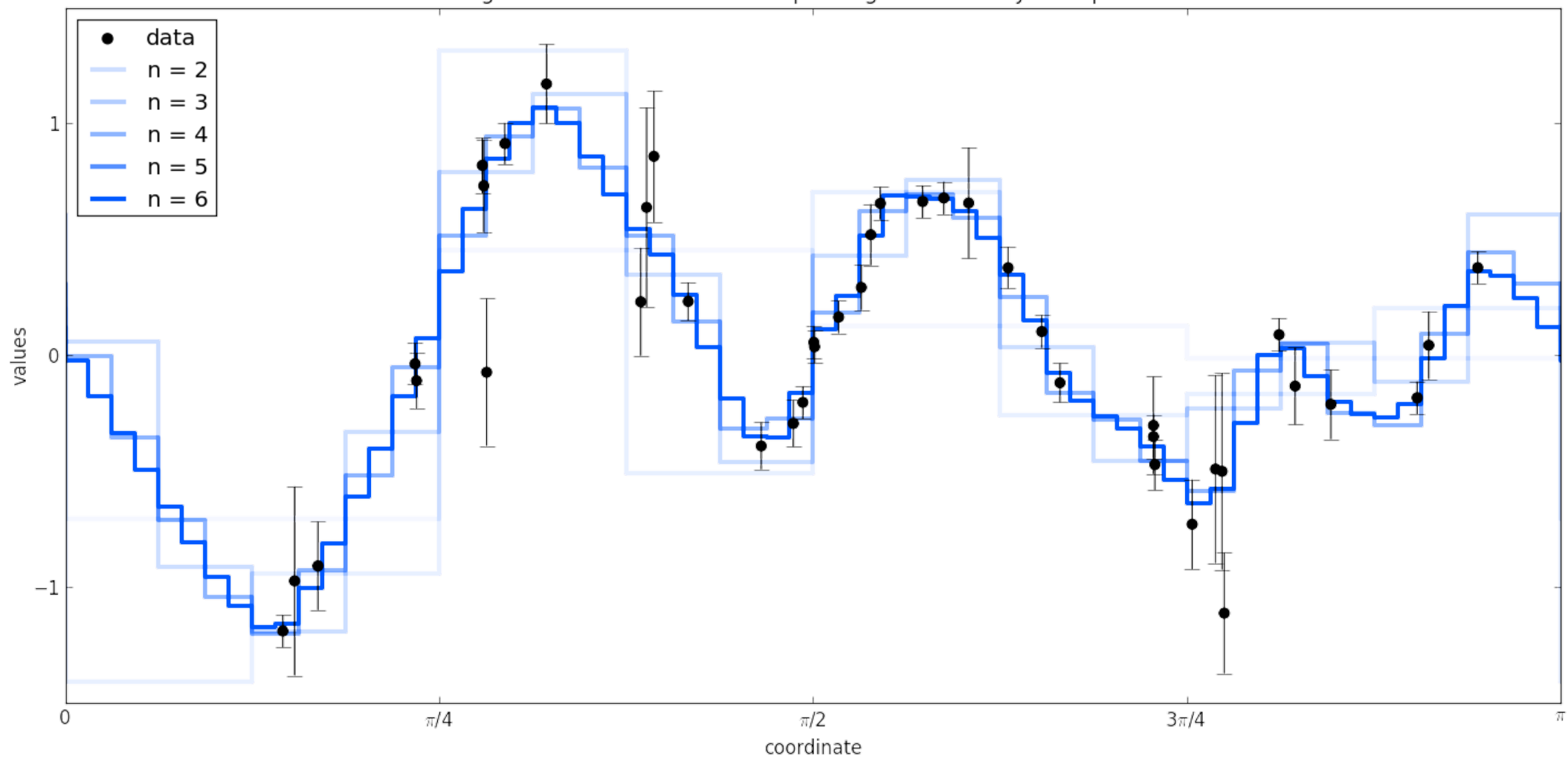
$$\mathcal{P}(s|d) \propto e^{-\frac{(s-m)^2}{2\sigma''^2}}$$

$$\mathcal{H}(d, s) \triangleq \frac{(s-m)^2}{2\sigma''^2}$$



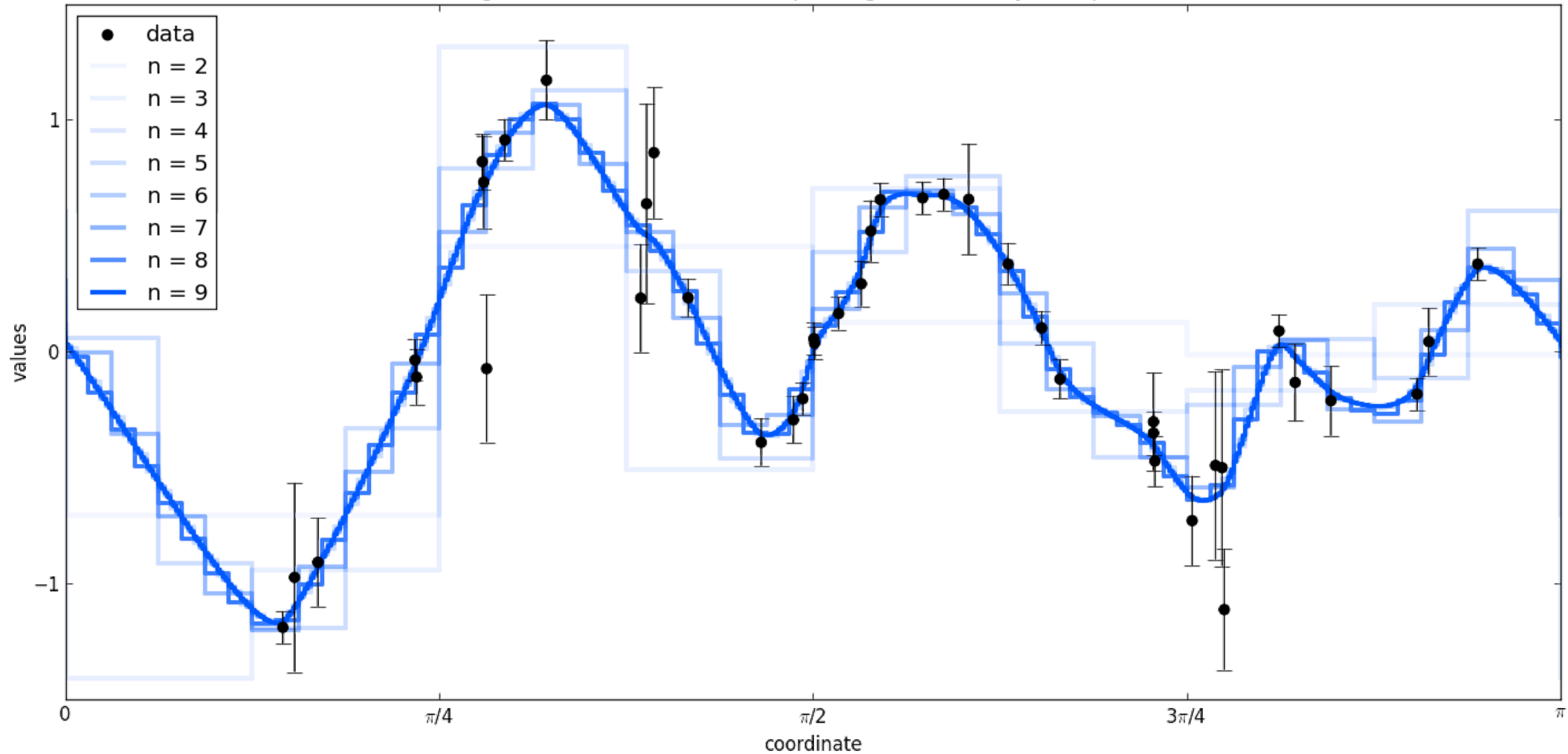


signal reconstruction with 2^n pixels given 42 noisy data points



Correlations

signal reconstruction with 2^n pixels given 42 noisy data points

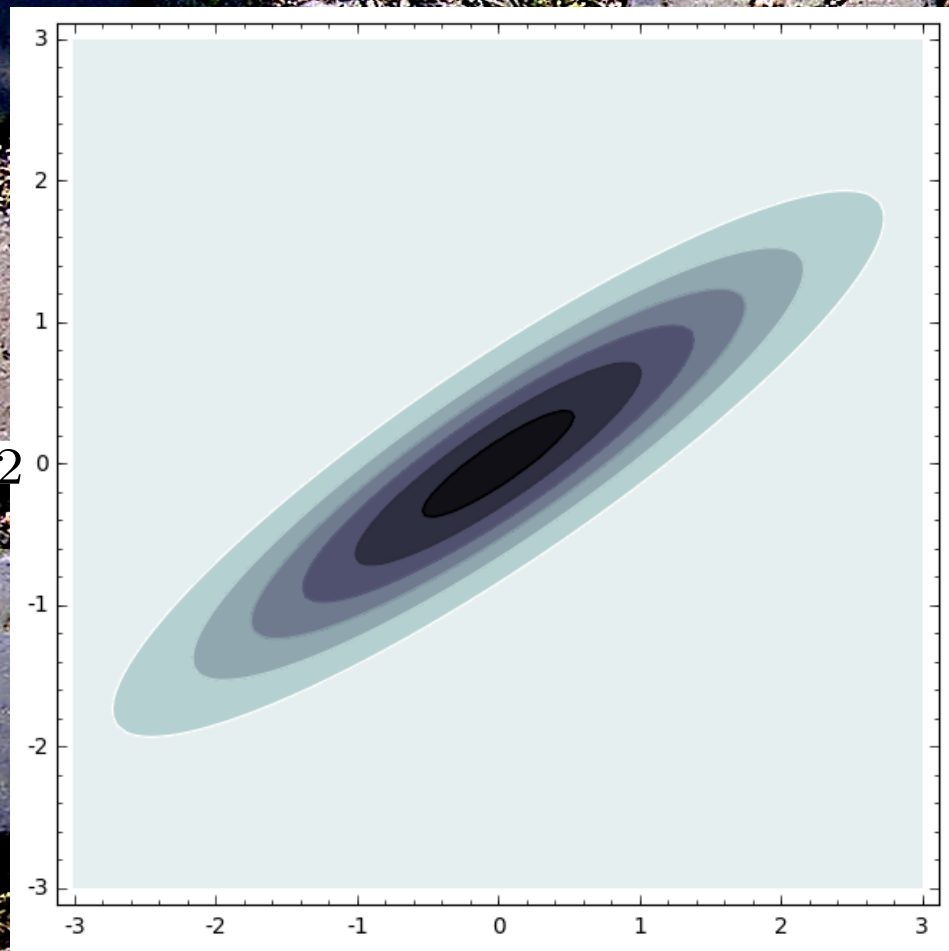


Correlations

$$\mathcal{P}(s)$$

$$s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$

$$d = s_1 + n$$



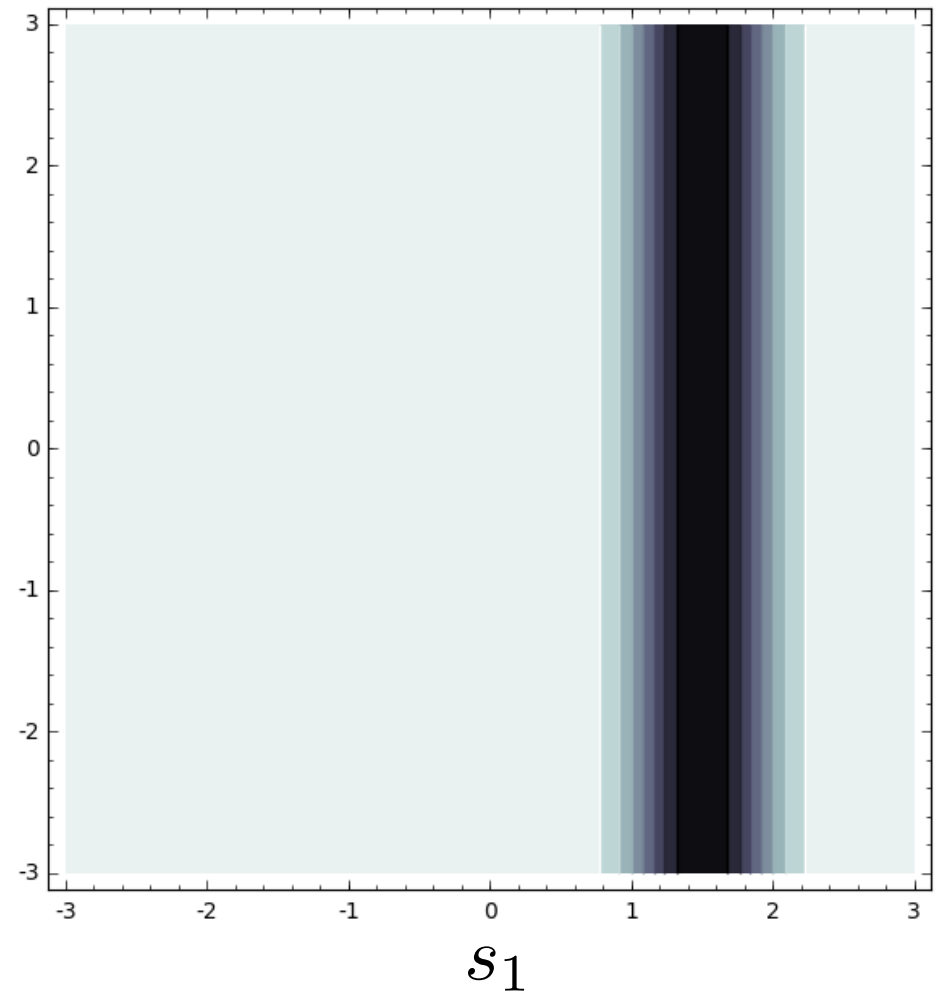
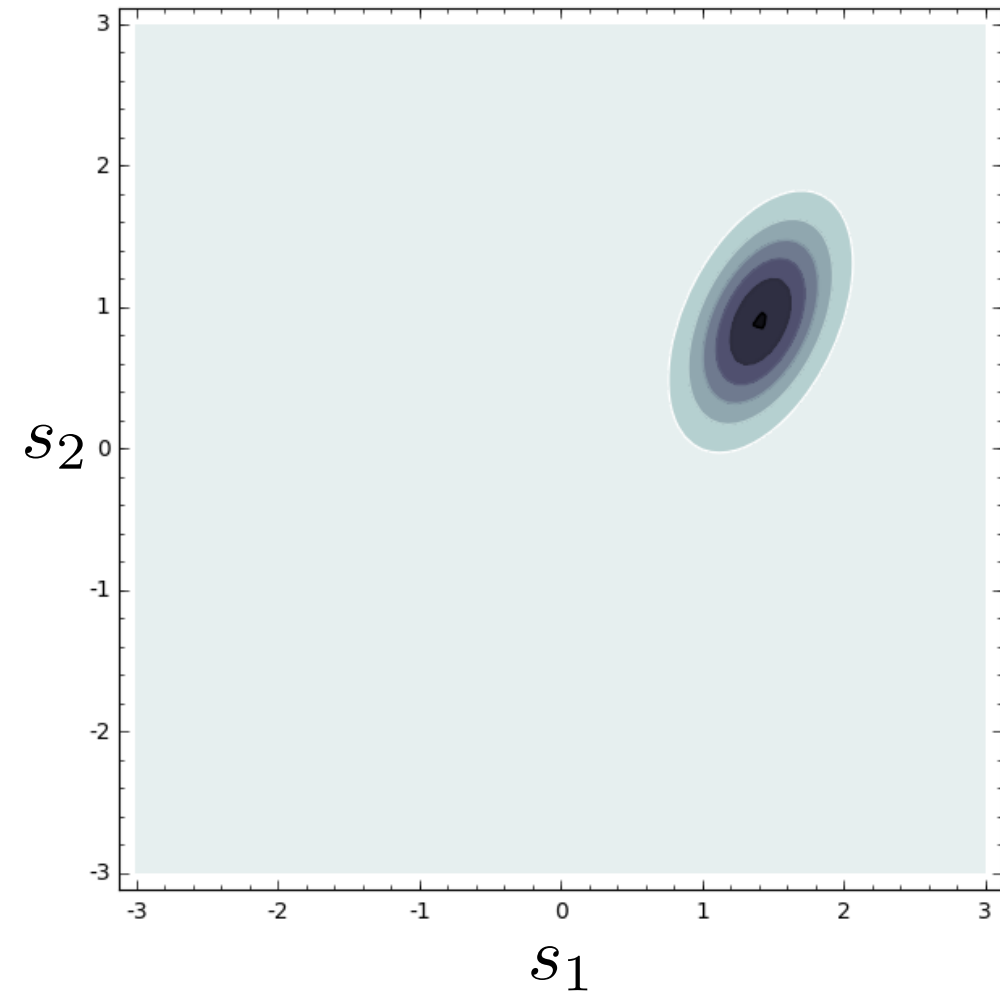
s_1

s_2

Correlations

$$\mathcal{P}(s|d) \quad s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$

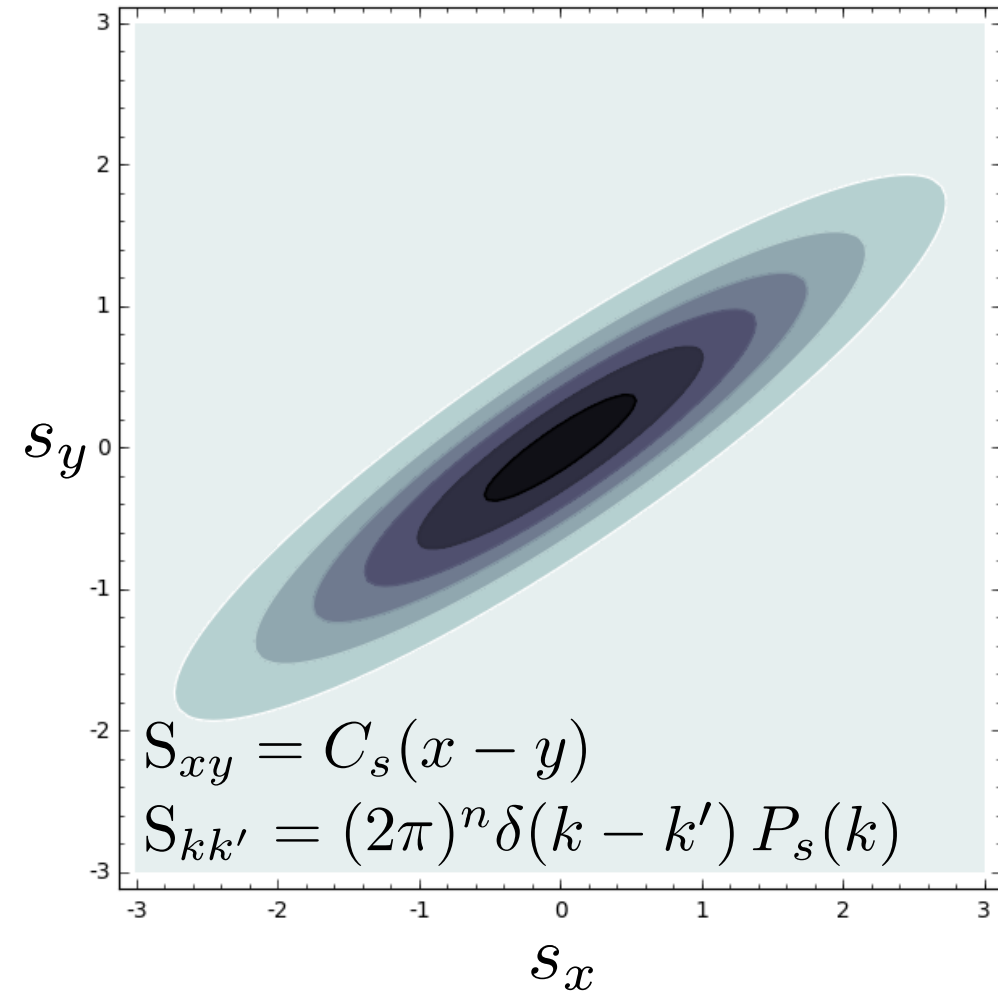
$$\mathcal{P}(d|s) \\ d = s_1 + n$$



Correlations

$$\mathcal{P}(s)$$

$$s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$



$$\mathcal{P}(s) = \mathcal{G}(s, S)$$

$$= \frac{1}{\sqrt{|2\pi S|}} \exp \left(-\frac{1}{2} s^\dagger S^{-1} s \right)$$

$$S = \begin{pmatrix} \langle s_1 s_1 \rangle & \langle s_1 s_2 \rangle \\ \langle s_2 s_1 \rangle & \langle s_2 s_2 \rangle \end{pmatrix} \quad \text{2-dim.}$$

$$S_{ij} = \langle s_i s_j \rangle \quad n\text{-dim.}$$

$$S_{xy} = \langle s_x s_y \rangle, \quad x \in \mathbb{R}^n \quad \infty\text{-dim.}$$

$$s^\dagger S^{-1} s = \int dx \int dy s_x (S^{-1})_{xy} s_y$$

$$s^\dagger j = \int dx s_x j_x$$

Wiener filter

$$d = R s + n$$

data

$$\mathcal{P}(d, s | R, S, N) = \mathcal{G}(s, S) \mathcal{G}(d - R s, N)$$

prior & likelihood

$$\mathcal{P}(s | d, R, S, N) = \mathcal{G}(s - m, D)$$

posterior

$$m = D j$$

posterior mean

$$j = R^\dagger N^{-1} d$$

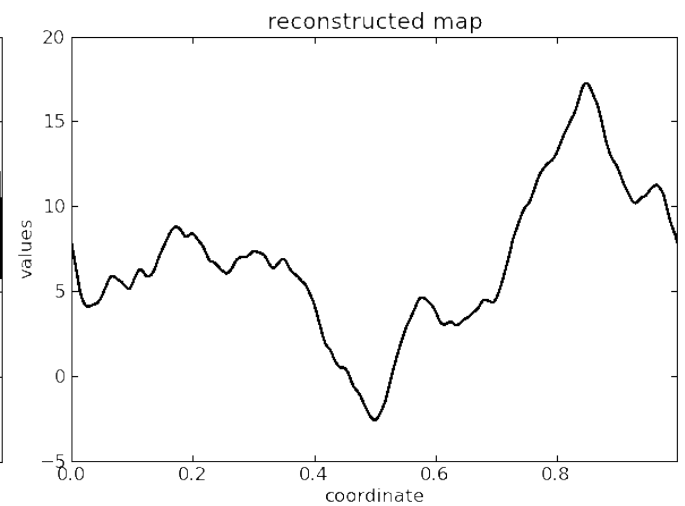
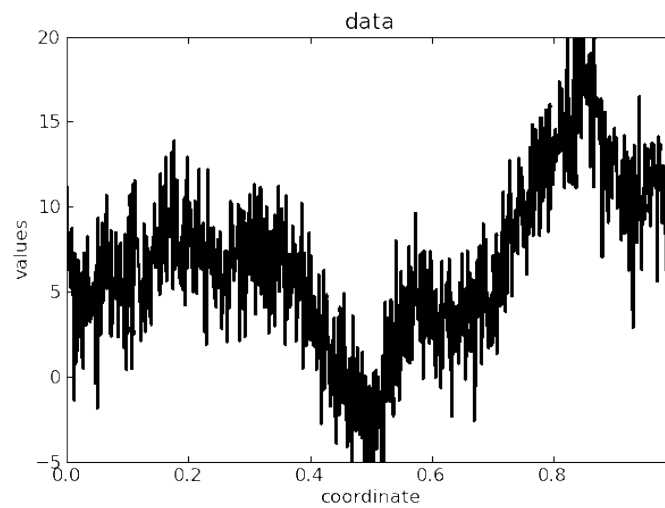
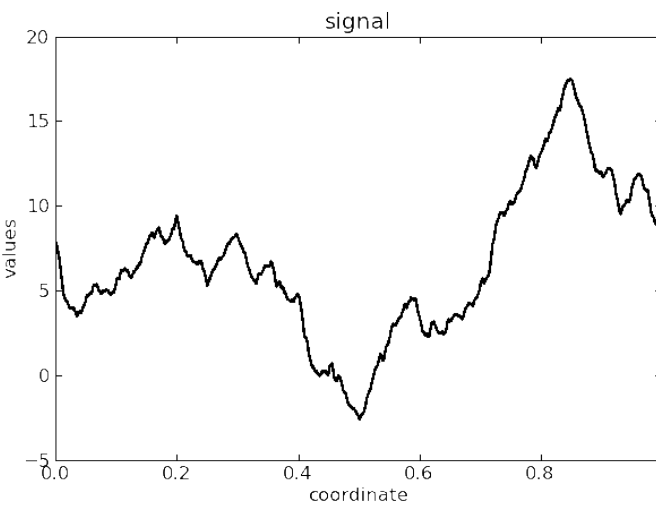
information source

$$D = (S^{-1} + R^\dagger N^{-1} R)^{-1}$$

information propagator

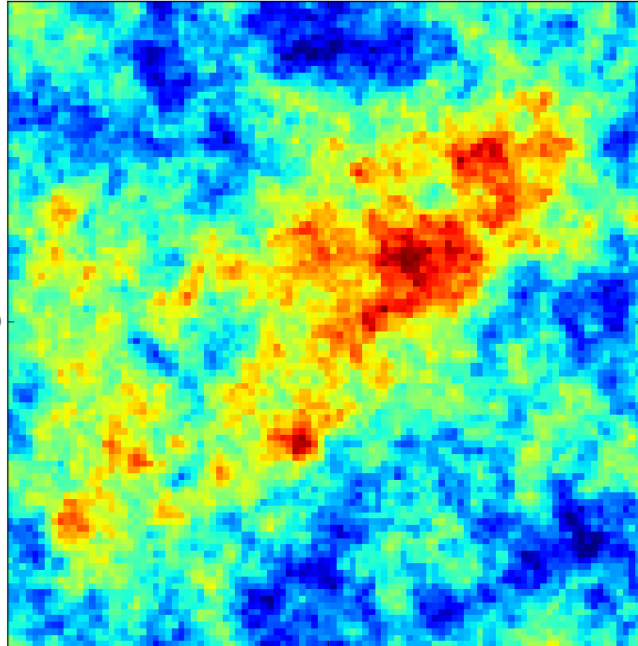


Wiener filter

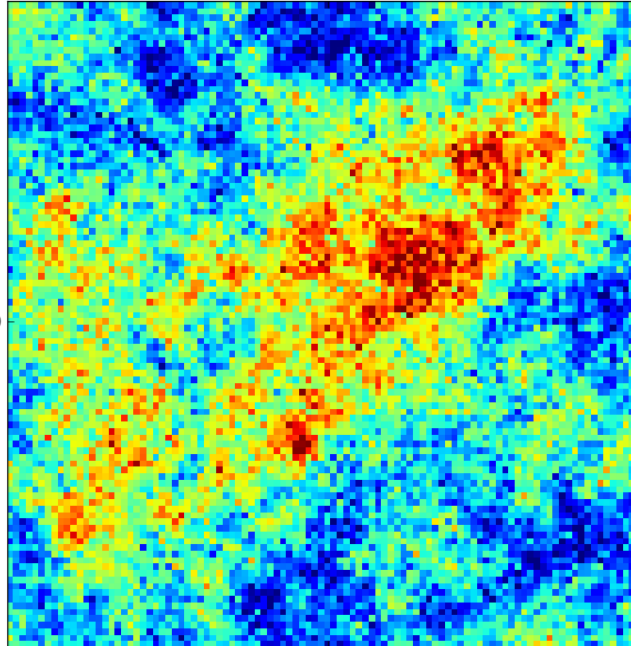


NIFTy – Numerical Information Field Theory

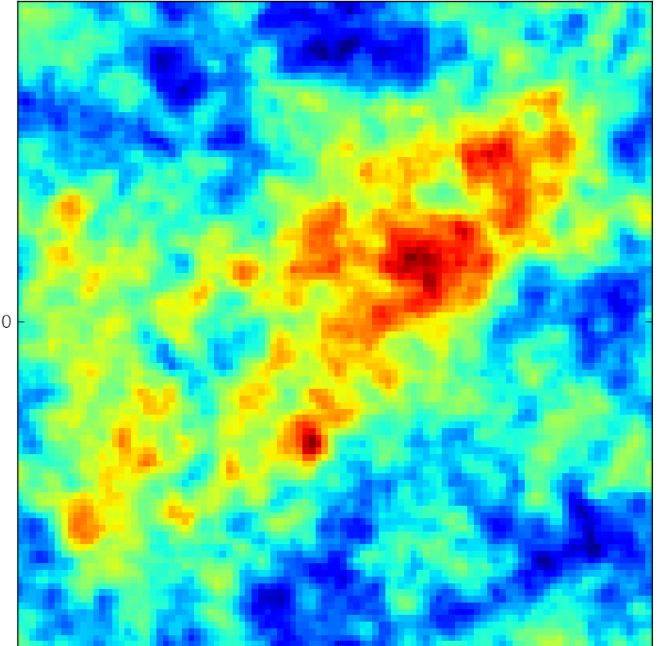
signal



data



reconstructed map



```
import nifty5 as ift
```

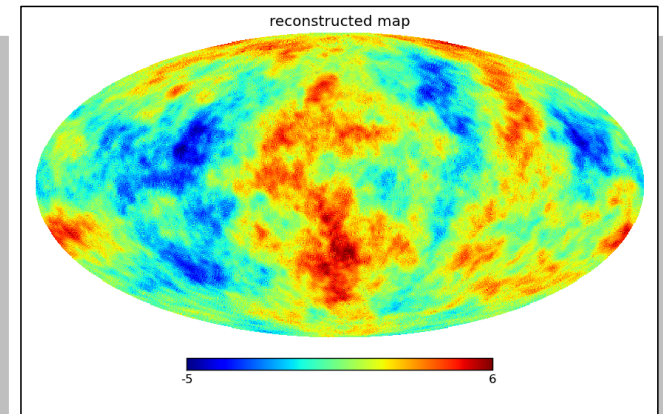
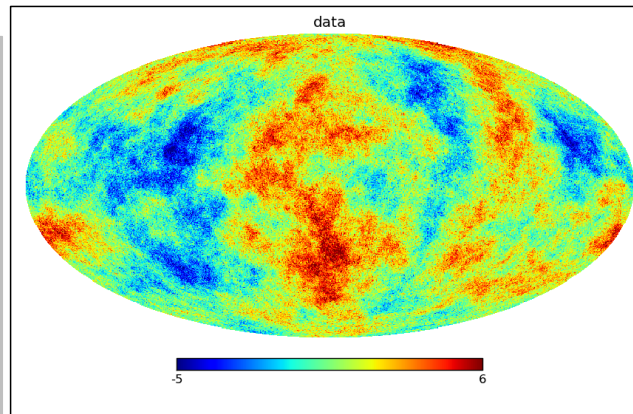
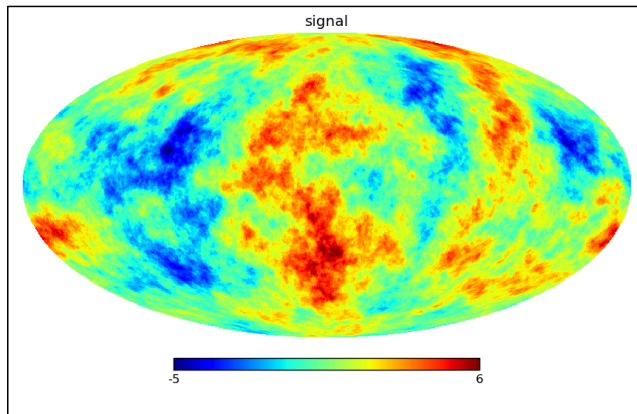
```
...
```

```
s_space = ift.HPSpace(NSide)
```

```
...
```

```
m = D(j)
```

NIFTy – Numerical Information Field Theory



```
import nifty5 as ift
```

```
...
```

```
s_space = ift.HPSpace(NSide)
```

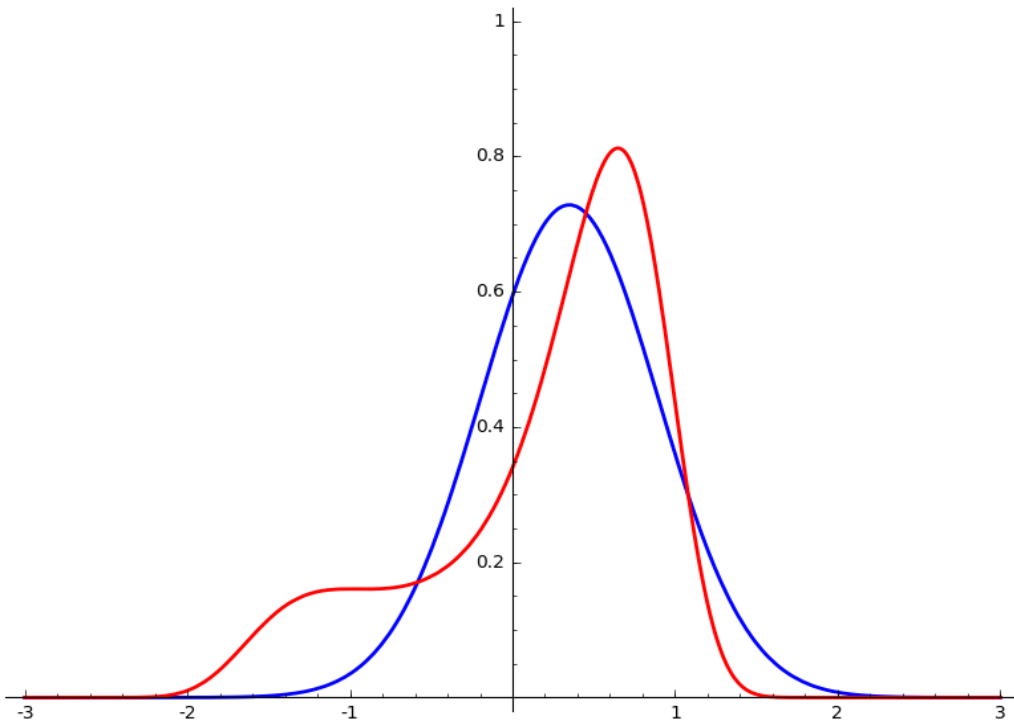
```
...
```

```
m = D(j)
```

Variational Bayes

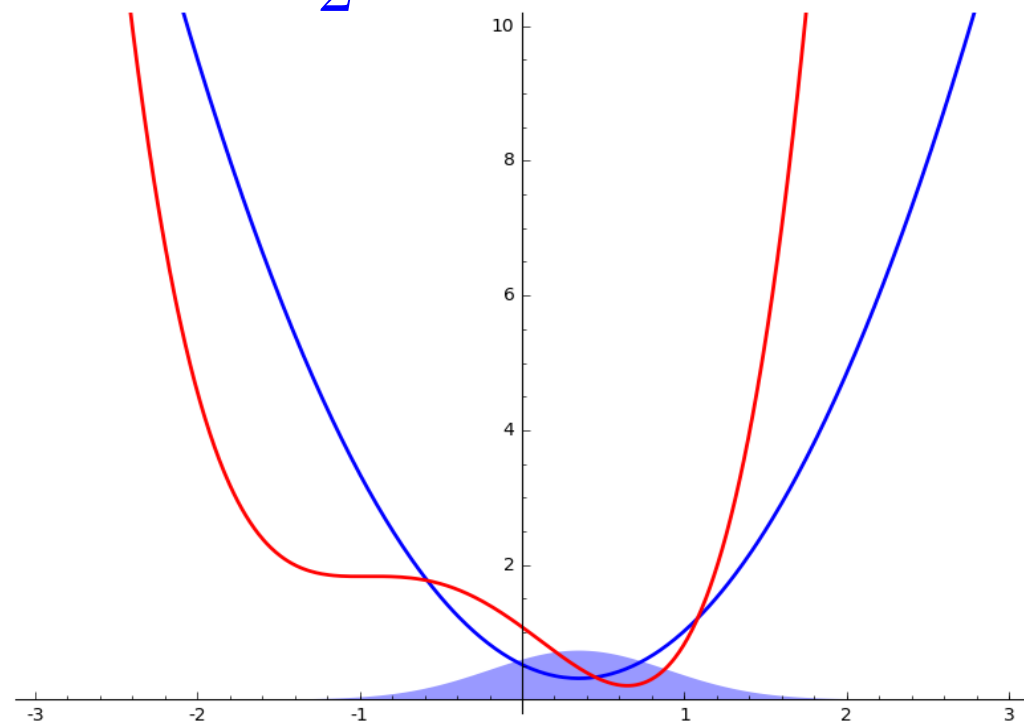
$$\mathcal{P}(s|d)$$

$$\tilde{\mathcal{P}}(s|d) = \mathcal{G}(s - m, D)$$



$$\mathcal{H}(s|d)$$

$$\tilde{\mathcal{H}}(s|d) \hat{=} \frac{1}{2} (s - m)^\dagger D^{-1} (s - m)$$



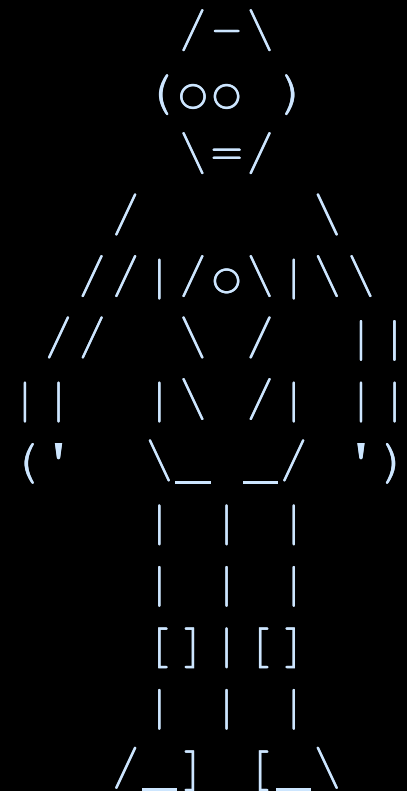
$$\text{KL}(\tilde{\mathcal{P}}, \mathcal{P}) = \int \mathcal{D}s \tilde{\mathcal{P}}(s|d) \left[\mathcal{H}(s|d) - \tilde{\mathcal{H}}(s|d) \right]$$

Photon Observations

Selig et al. (2014)

www.mpa-garching.mpg.de/ift/d3po

D3PO



Data model

$$d = R e^{\mathbf{s}} + n$$

known

known response

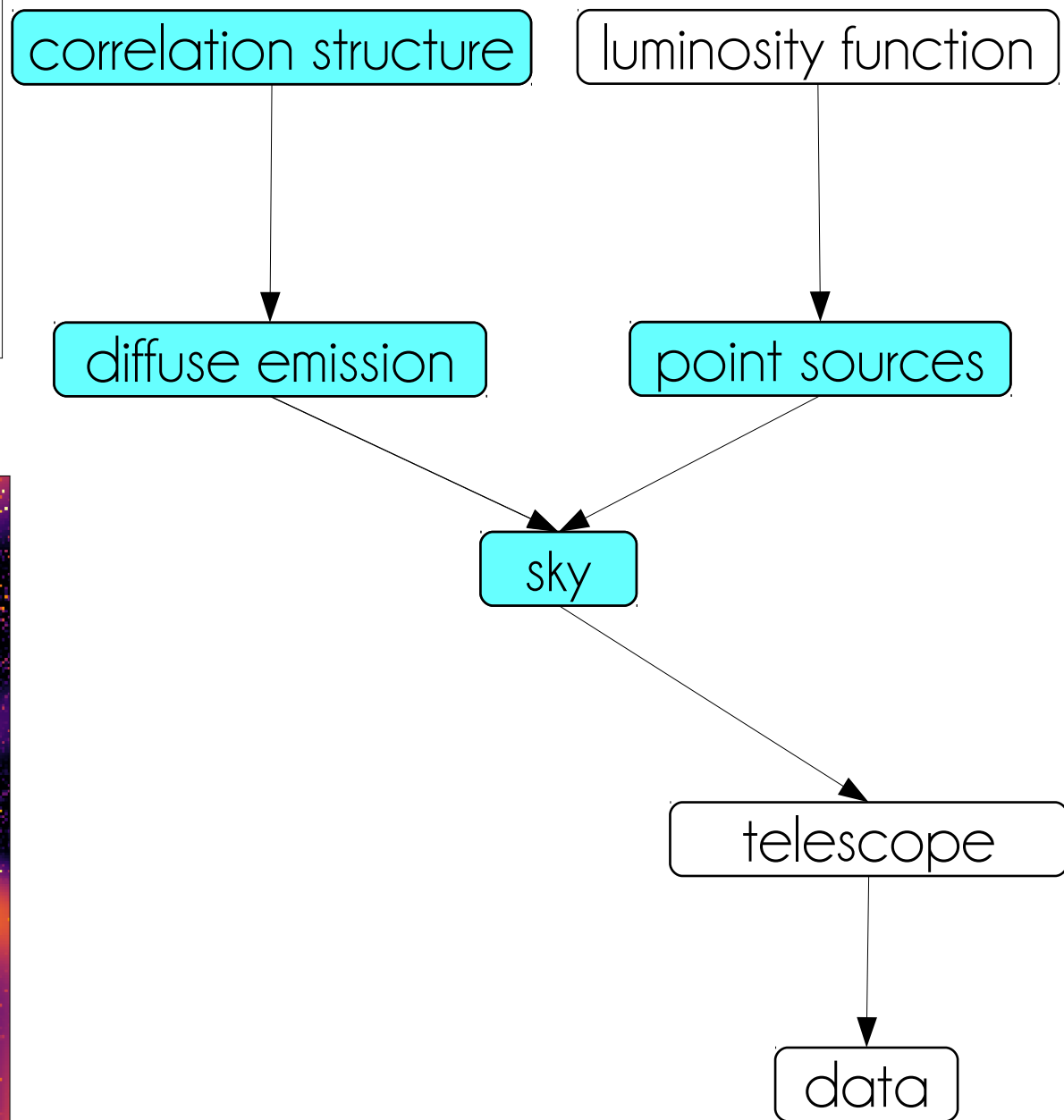
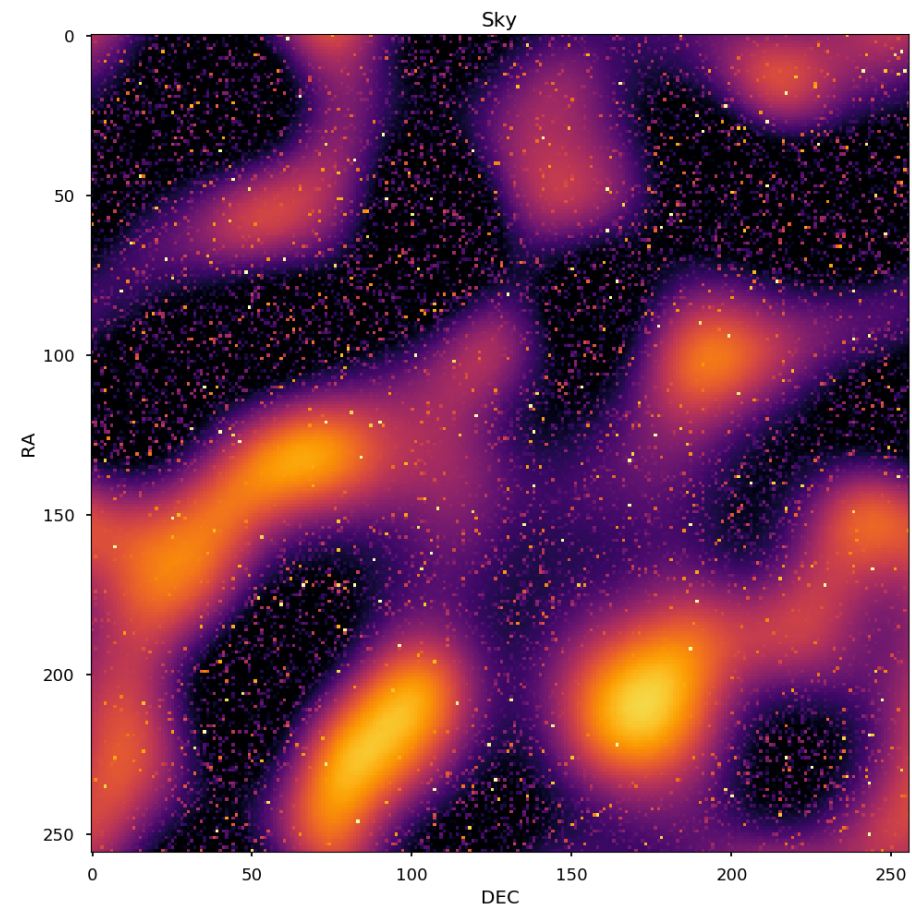
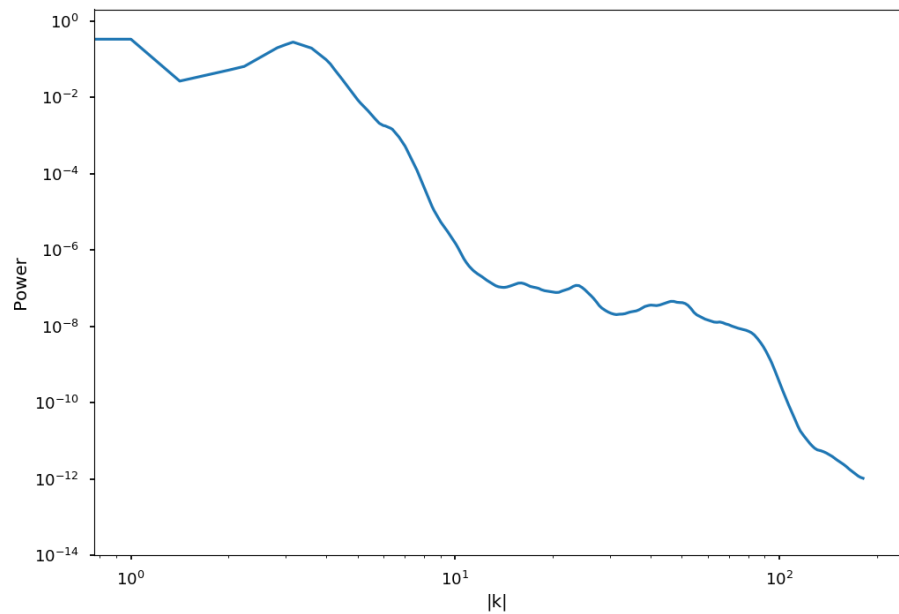
unknown

$$\lambda = R e^{\mathbf{s}}$$



$$\mathcal{P}(s) = \mathcal{G}(s, \mathbf{S}) \quad \text{unknown}$$

$$\mathcal{P}(d|\lambda) = \prod_i \frac{\lambda_i^{d_i}}{d_i!} e^{-\lambda_i}$$



Information

$$\mathcal{H}(\mathbf{d}, \mathbf{s}, \boldsymbol{\tau}) = -\log \mathcal{P}(\mathbf{d}, \mathbf{s}, \boldsymbol{\tau})$$

$$\text{likelihood} = 1^\dagger [\log(d!) + \mathbf{R} (e^{\mathbf{s}} + e^{\mathbf{u}})] - \mathbf{d}^\dagger \log [\mathbf{R} (e^{\mathbf{s}} + e^{\mathbf{u}})]$$

$$\begin{array}{l} \text{prior /} \\ \text{regularization} \\ \text{hyper-prior /} \\ \text{intelligence} \end{array} \quad + \frac{1}{2} \mathbf{s}^\dagger \mathbf{S}^{-1} \mathbf{s} + \frac{1}{2} \log (\det [\mathbf{S}])$$

$$+ (\boldsymbol{\alpha} - \mathbf{1})^\dagger \boldsymbol{\tau} + \mathbf{q}^\dagger e^{-\boldsymbol{\tau}} + \frac{1}{2} \boldsymbol{\tau}^\dagger \mathbf{T} \boldsymbol{\tau}$$

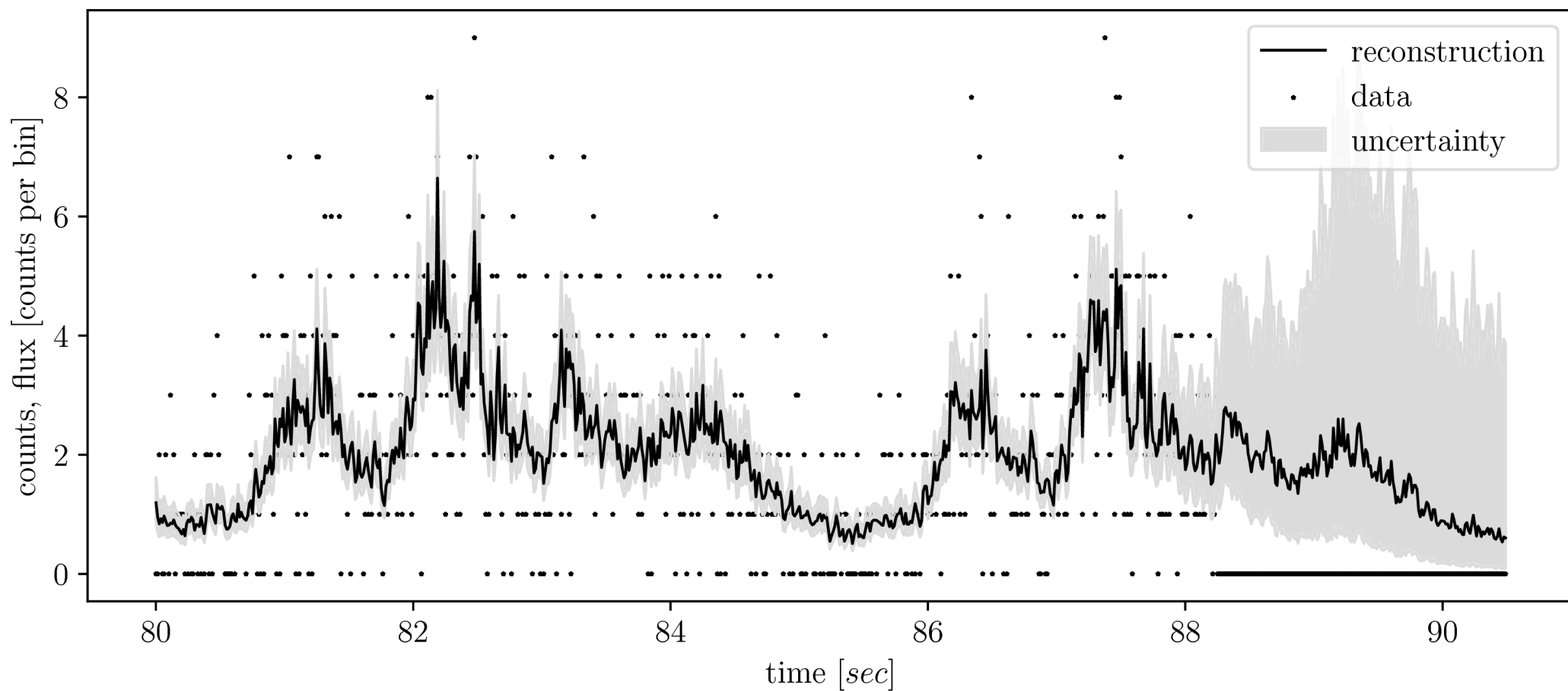
$$\text{point prior} \quad + (\boldsymbol{\beta} - \mathbf{1})^\dagger \mathbf{u} + \boldsymbol{\eta}^\dagger e^{-\mathbf{u}}$$

$$\begin{array}{l} \text{correlation} \\ \text{structure} \end{array} \quad \mathbf{S} = \sum_k e^{\tau_k} \mathbf{S}_k$$

D³PO in 1D & QPOs

Magnetar flare SGR 1900+14

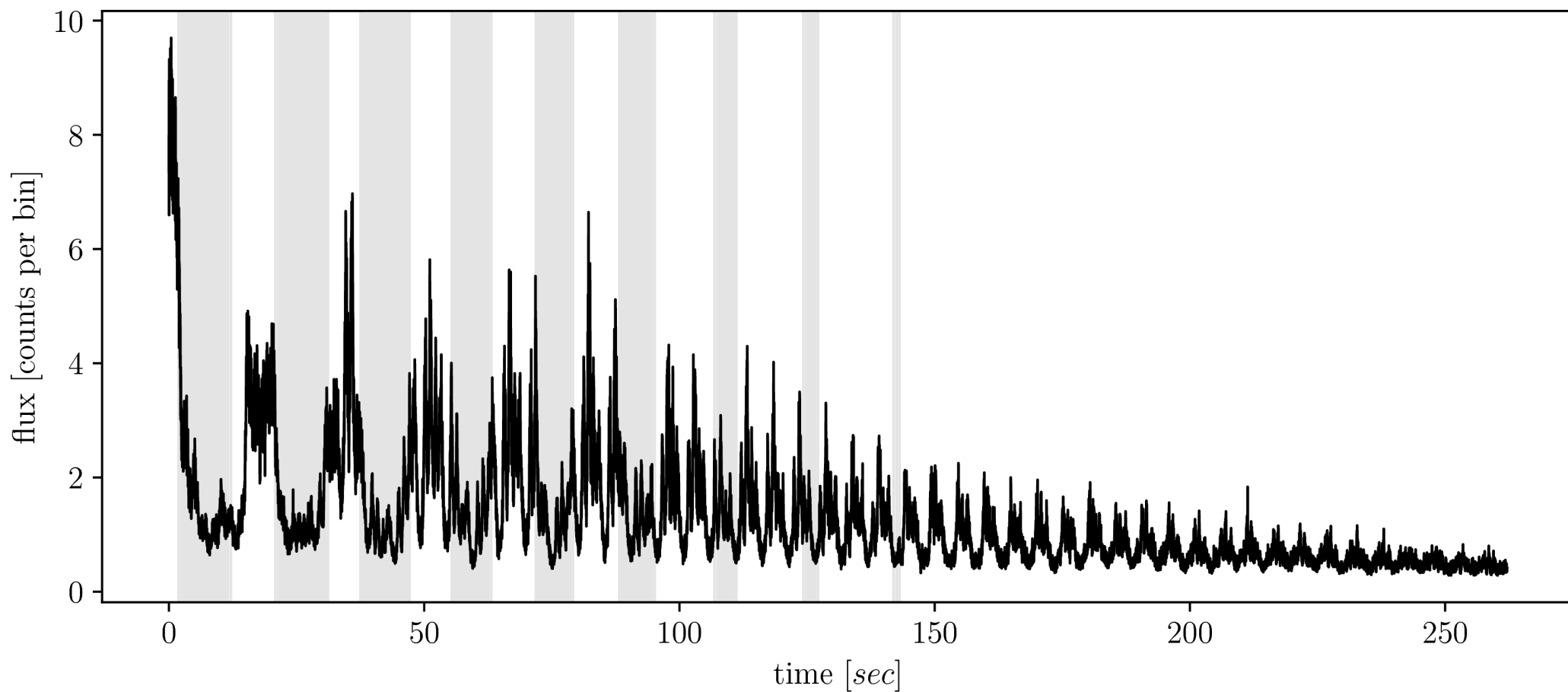
Pumpe et al. arXiv:1708.05702



D³PO in 1D & QPOs

Magnetar flare SGR 1900+14

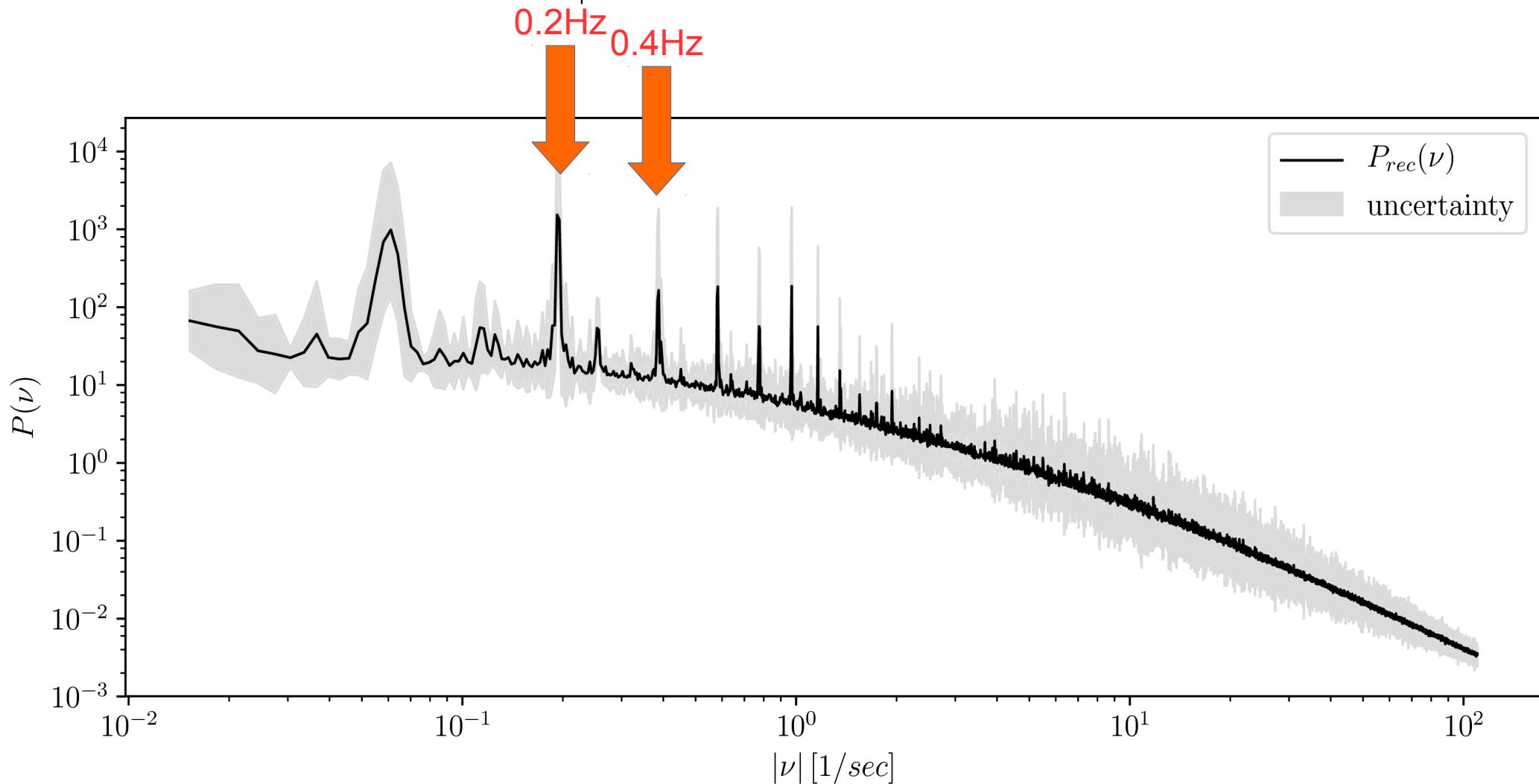
Pumpe et al. arXiv:1708.05702

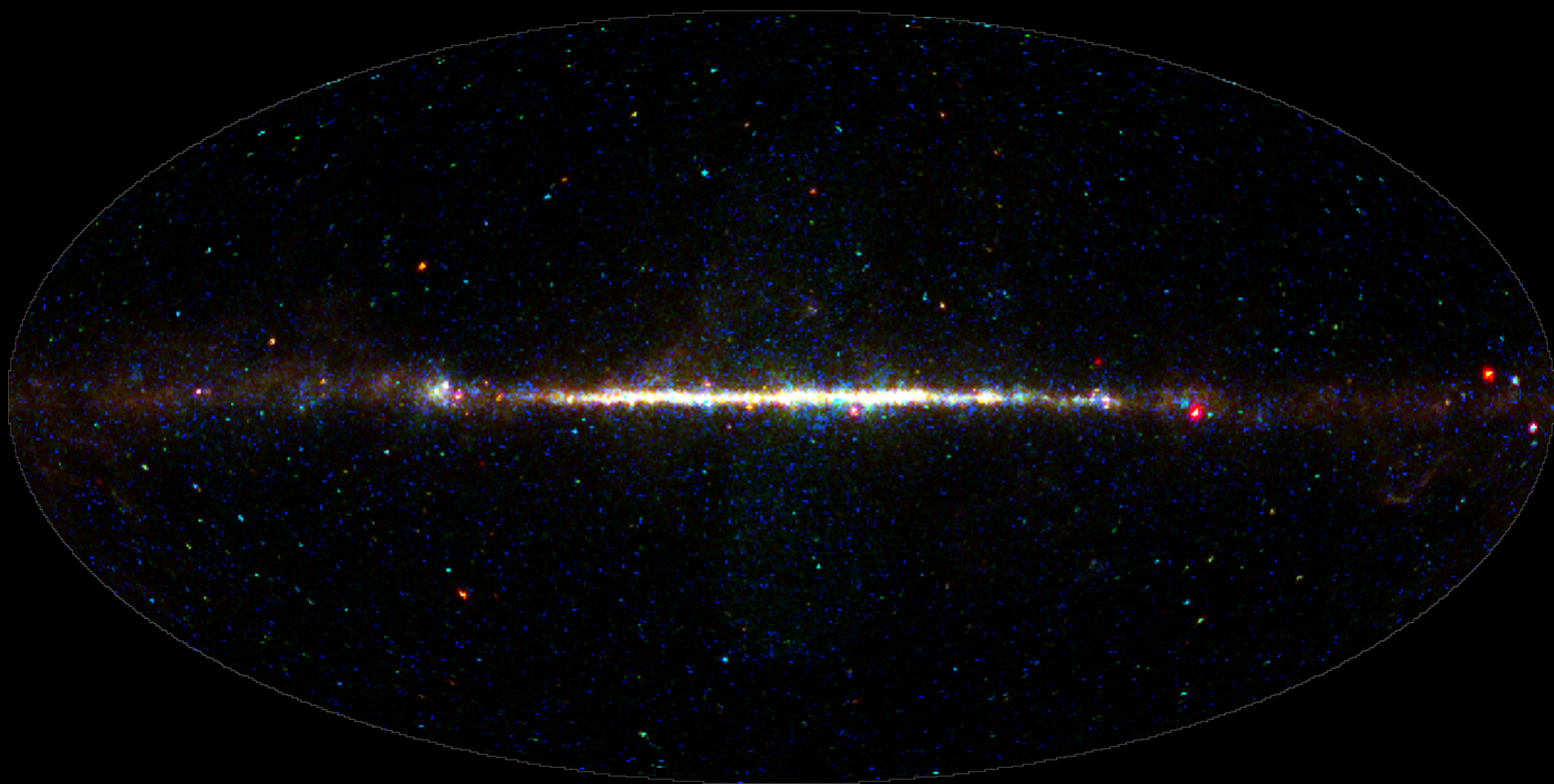


D³PO in 1D & QPOs

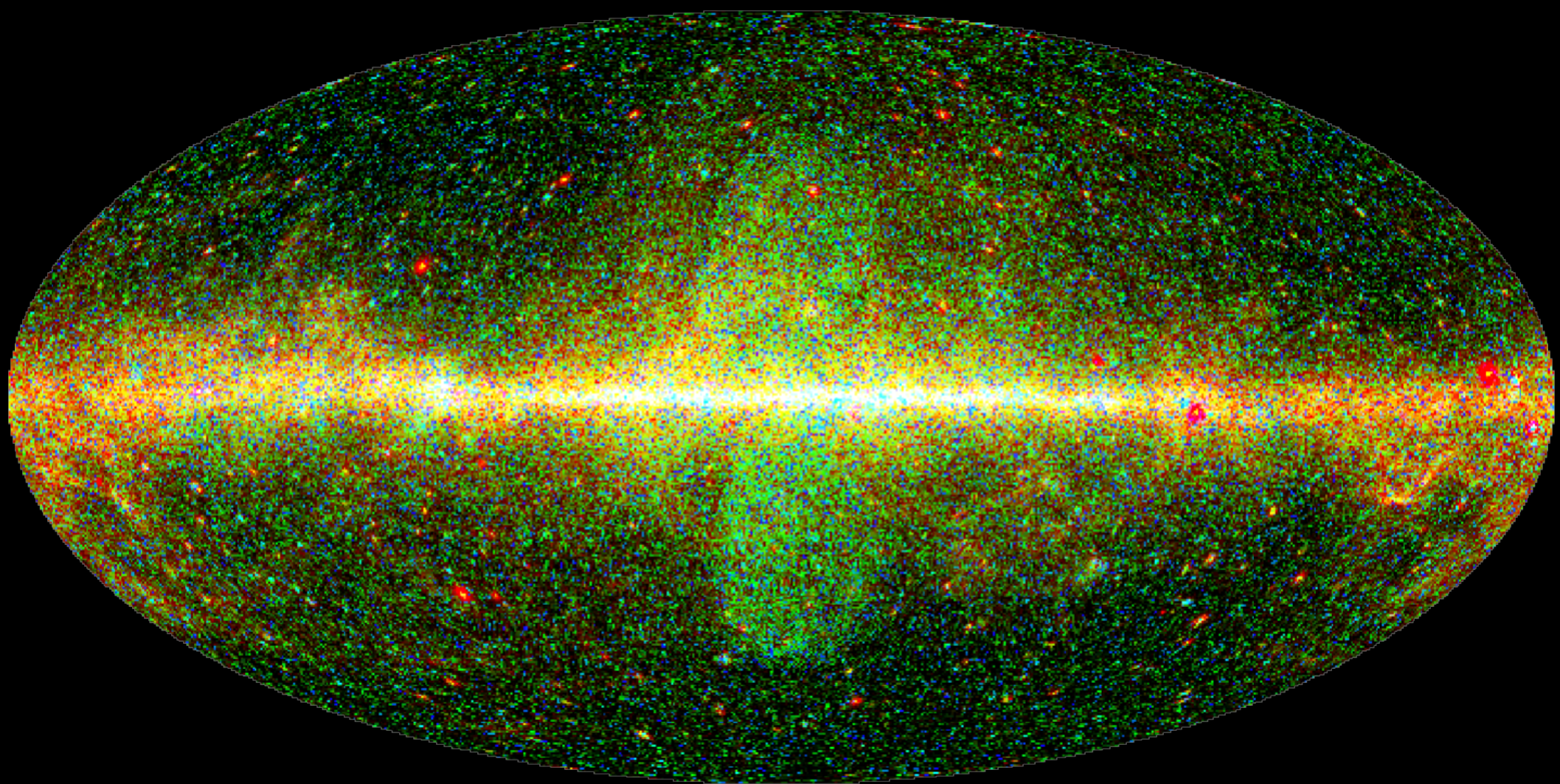
Magnetar flare SGR 1900+14

Pumpe et al. arXiv:1708.05702

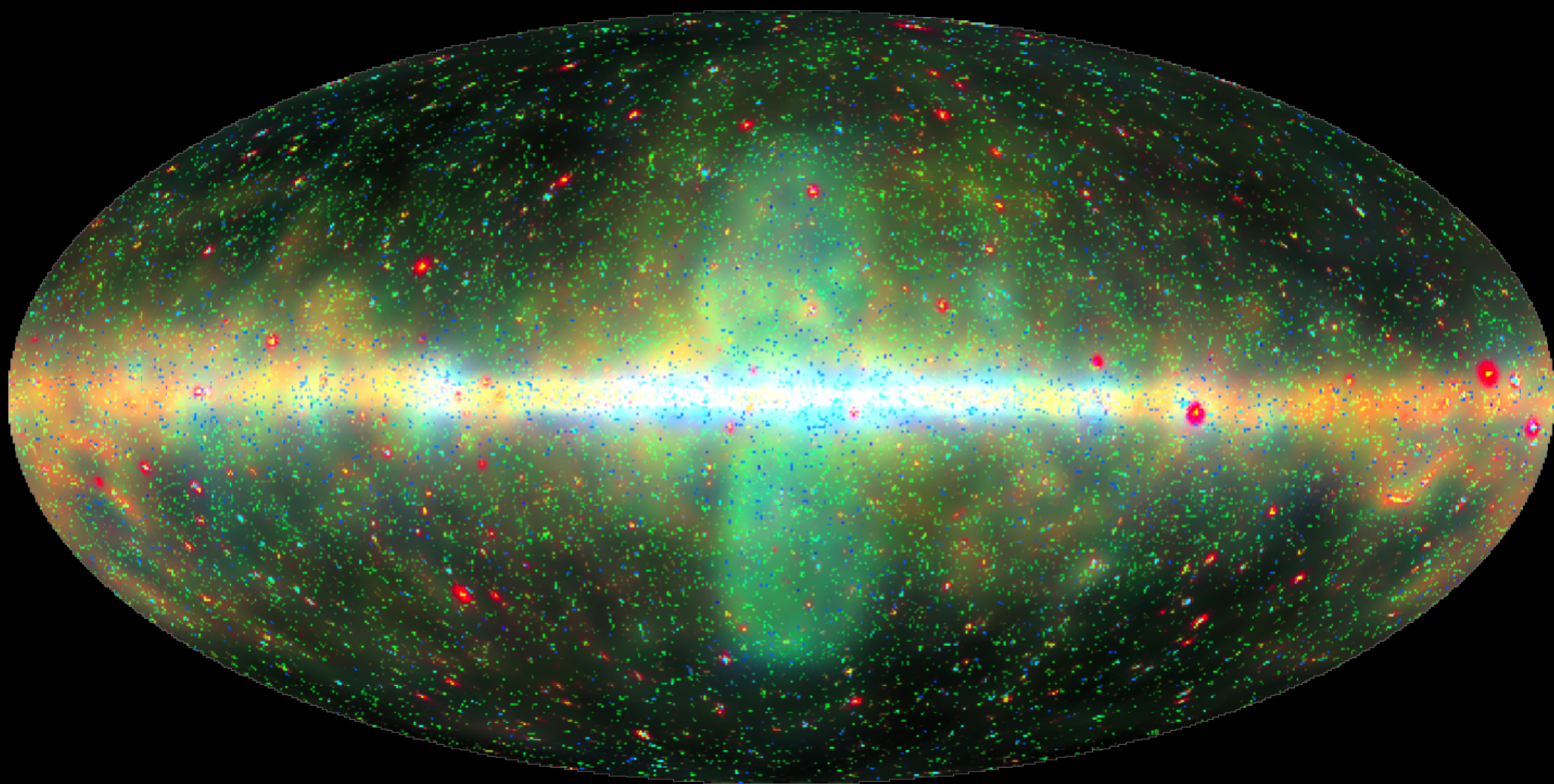




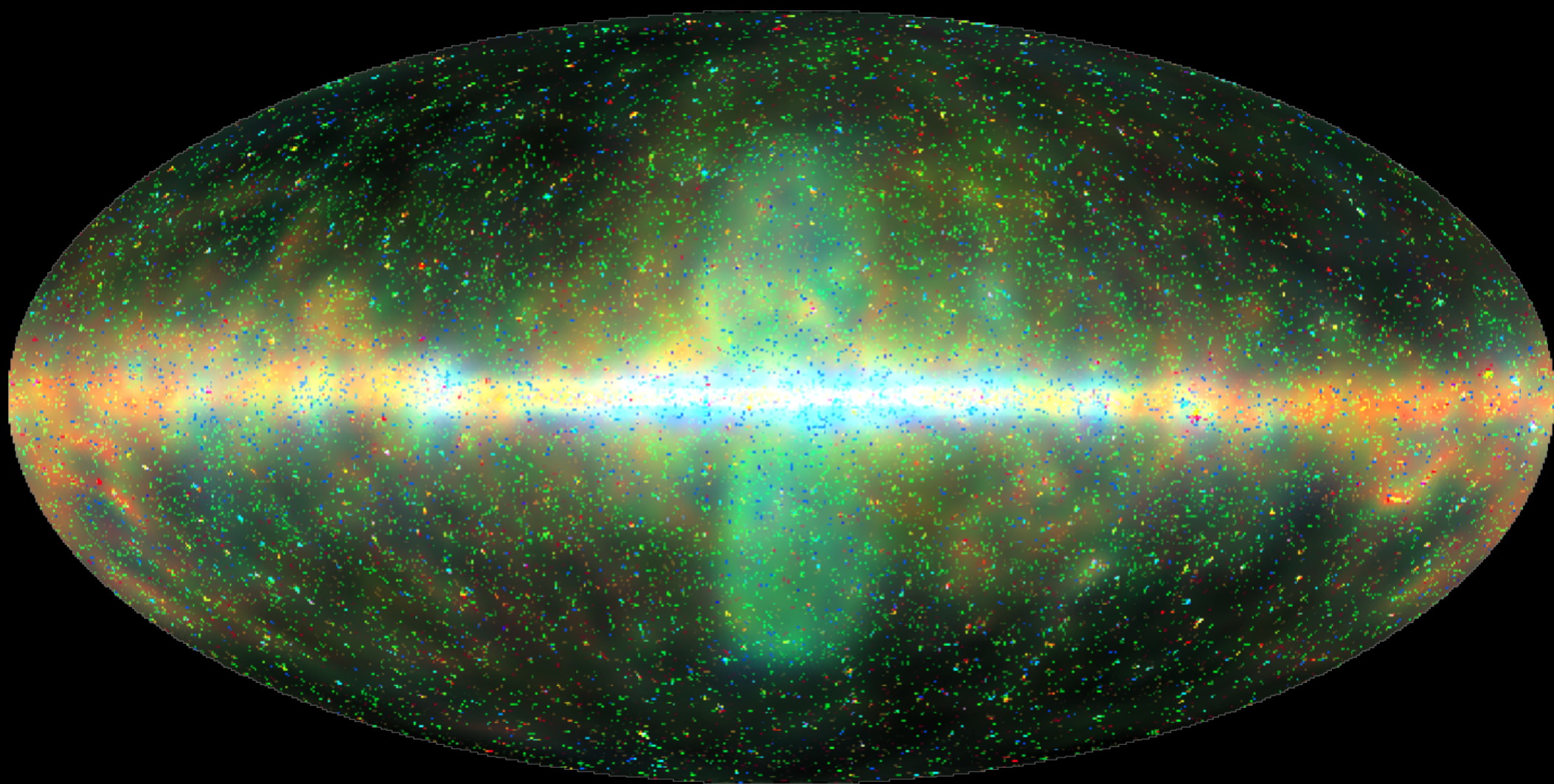
data



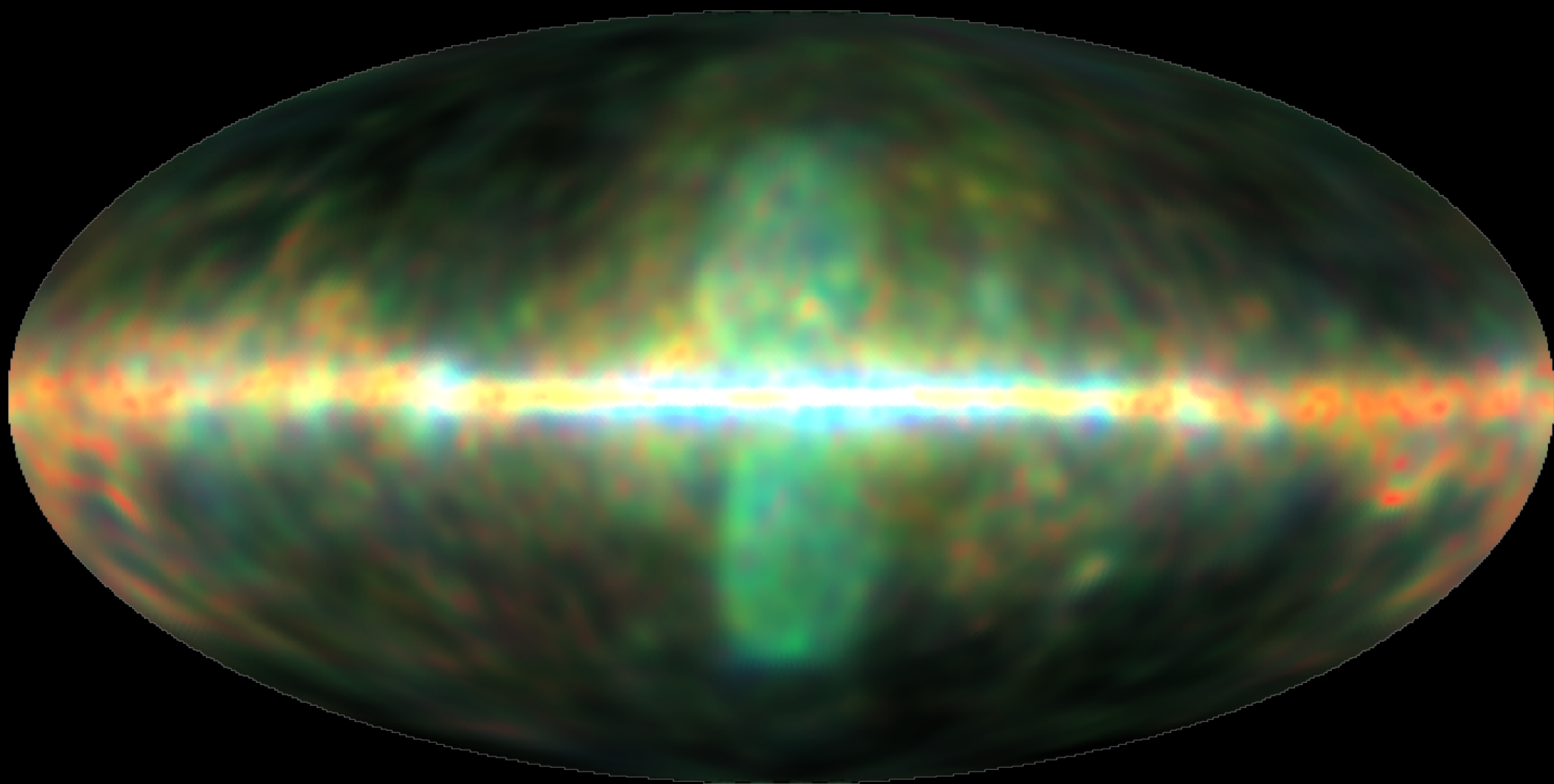
log-data



log-data ... denoised

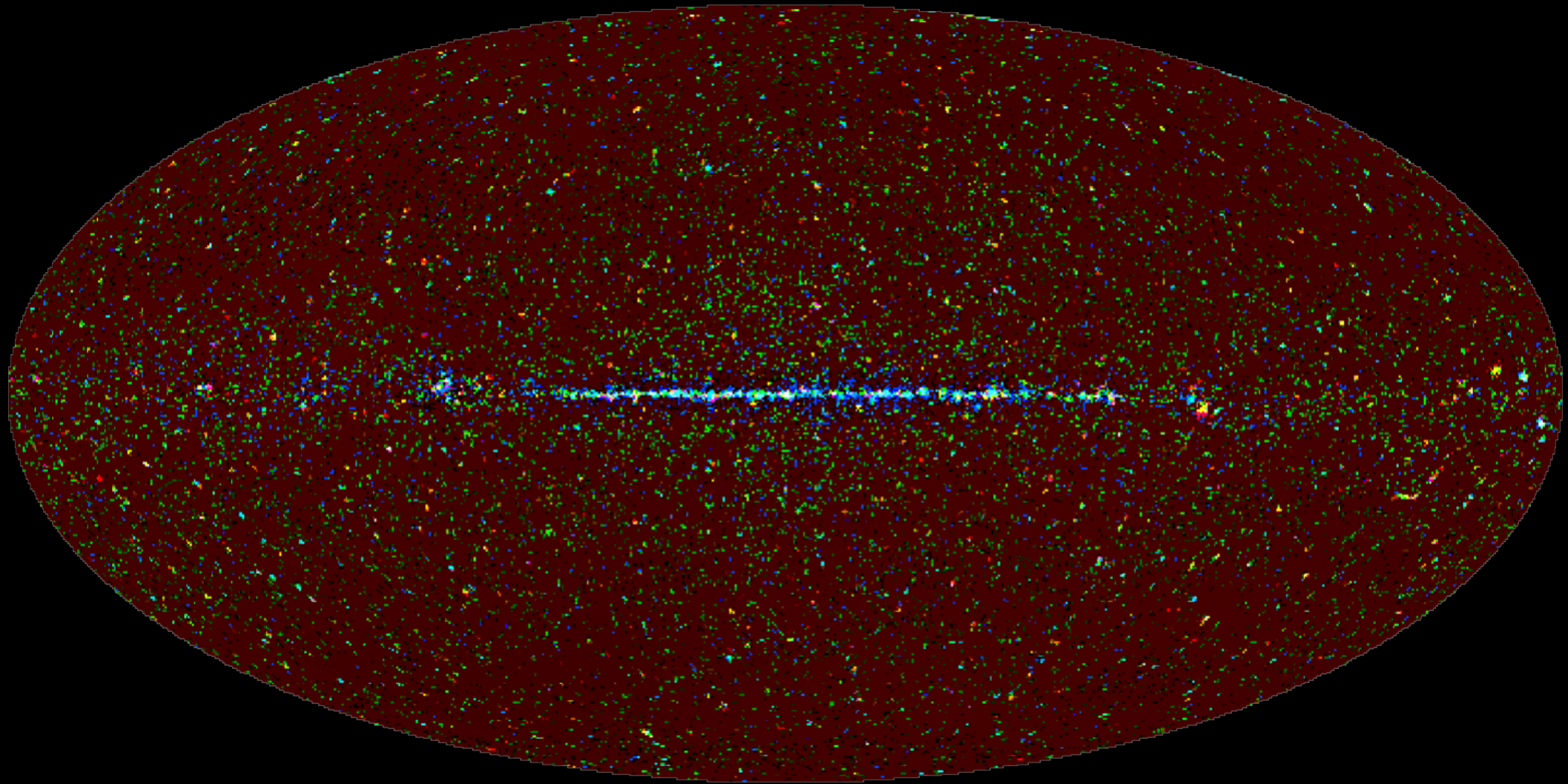


log-data ... denoised ... deconvolved



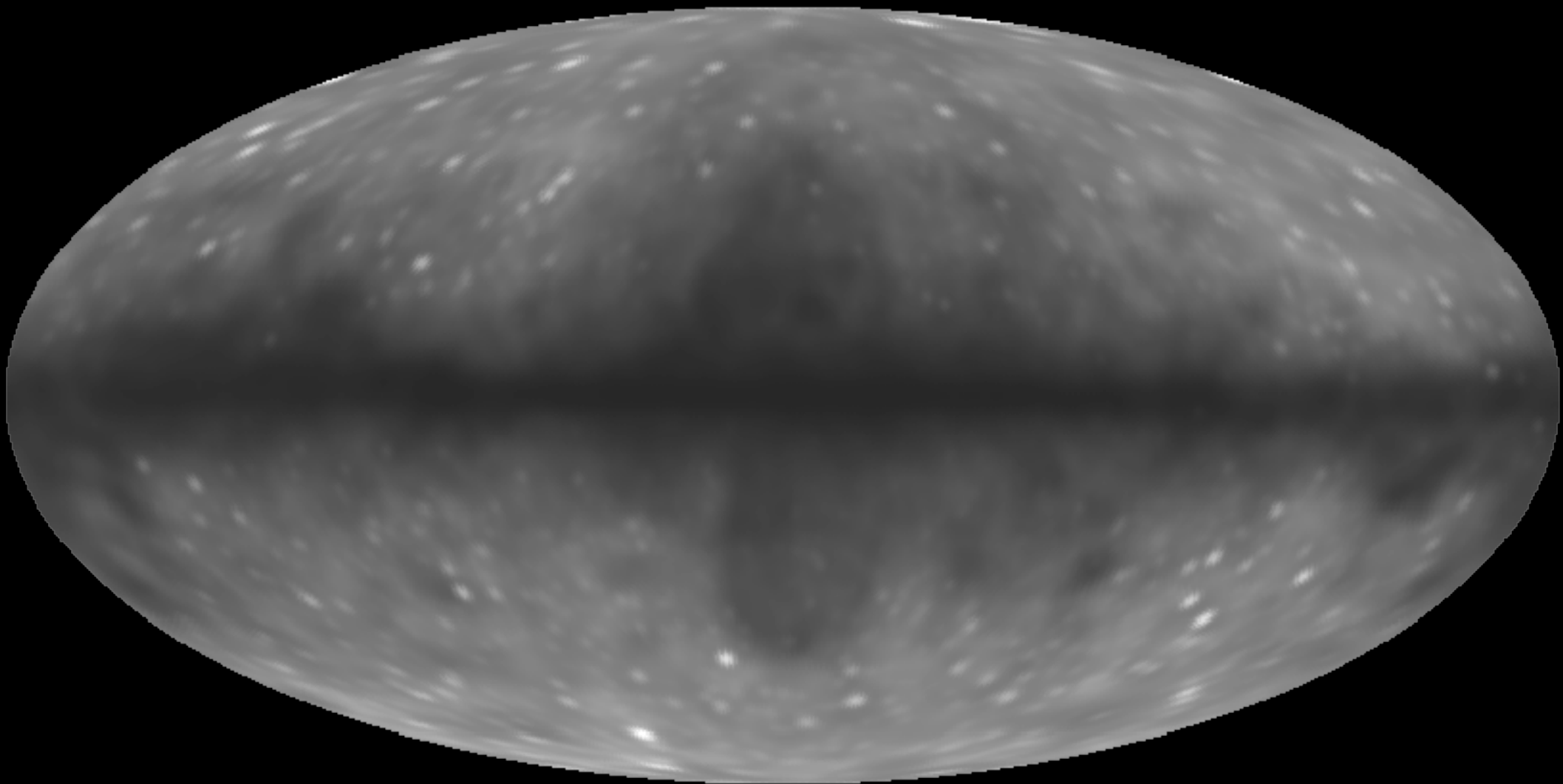
log-data ... denoised ... deconvolved ... decomposed

Selig, Vacca, Oppermann, Enßlin (2015)



log-data ... denoised ... deconvolved ... decomposed

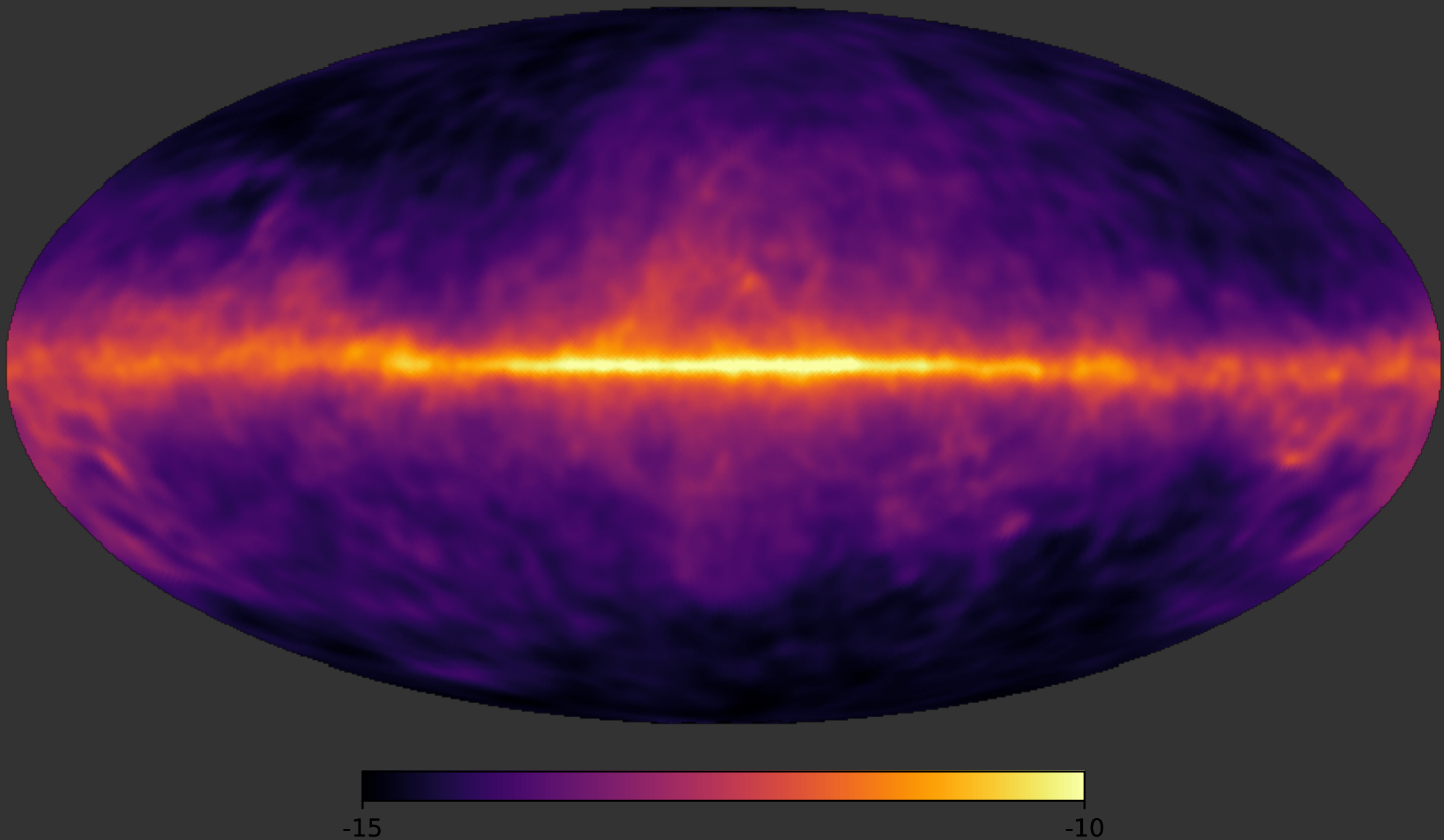
Selig, Vacca, Oppermann, Enßlin (2015)



relative uncertainty of diffuse emission

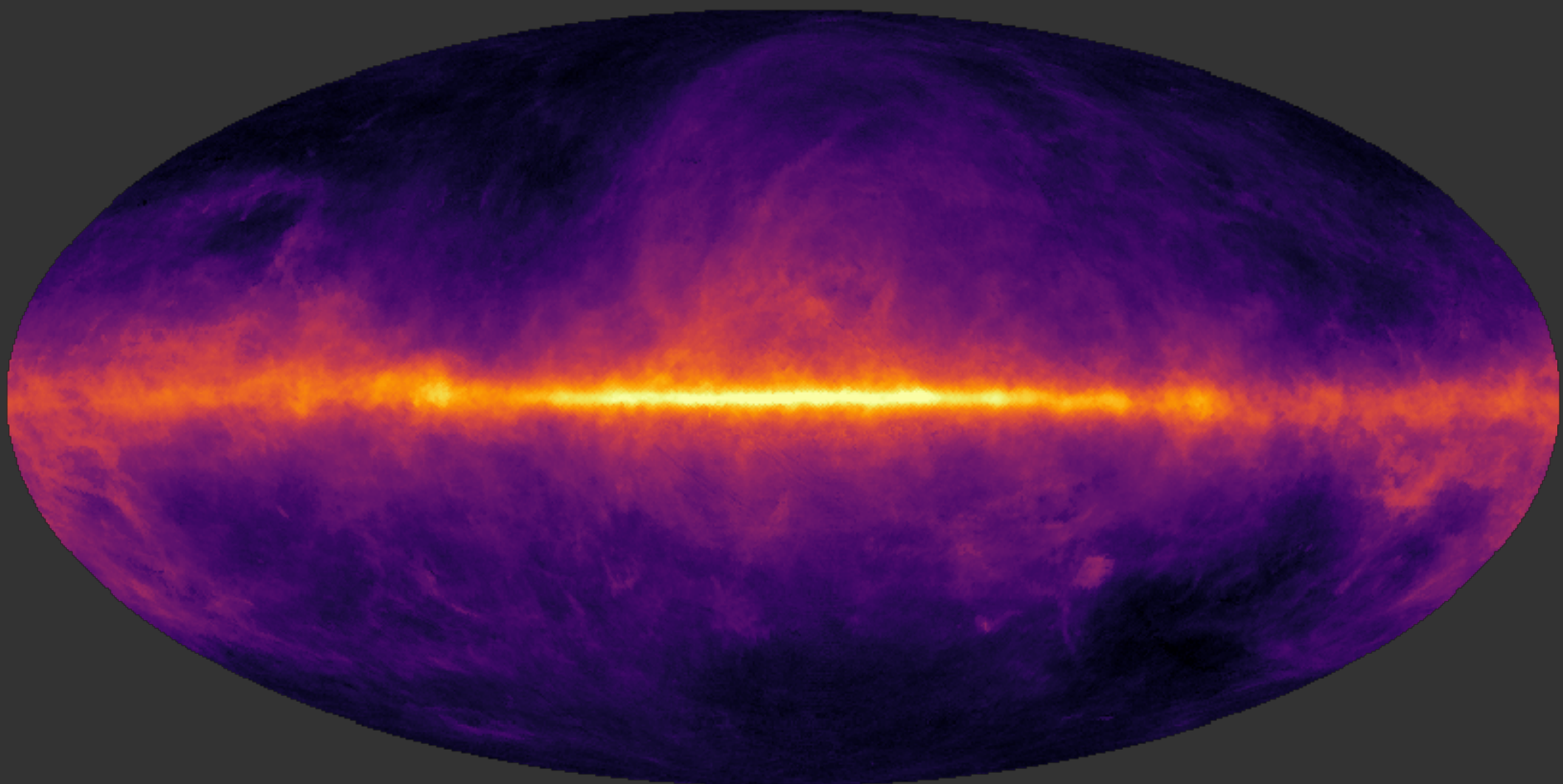
Sharpening up Galactic all-sky maps with complementary data

Ancla Müller et al (2018)



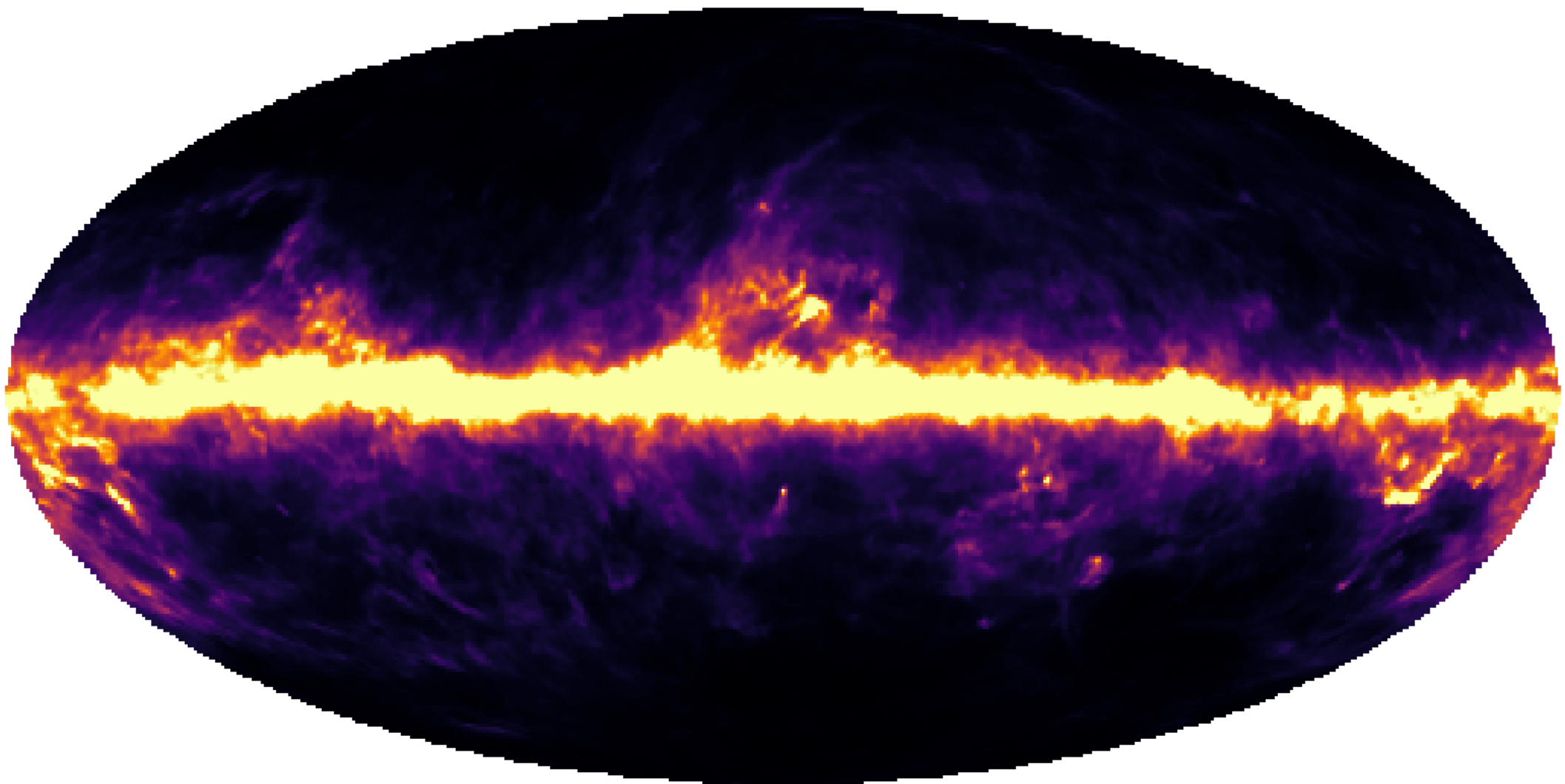
Sharpening up Galactic all-sky maps with complementary data

Ancla Müller et al (2018)



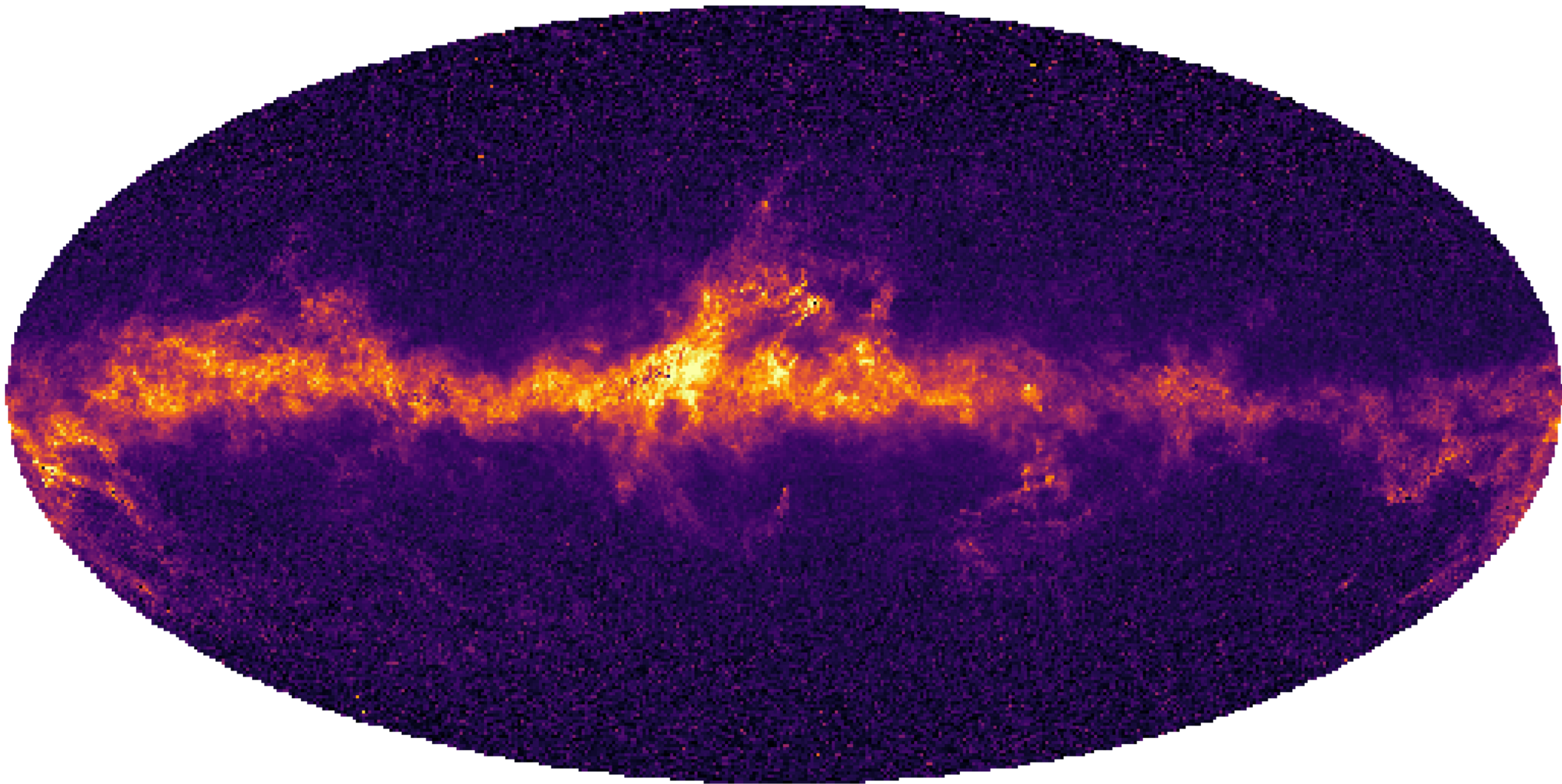
Planck Dust

Leike & Enßlin (2019)



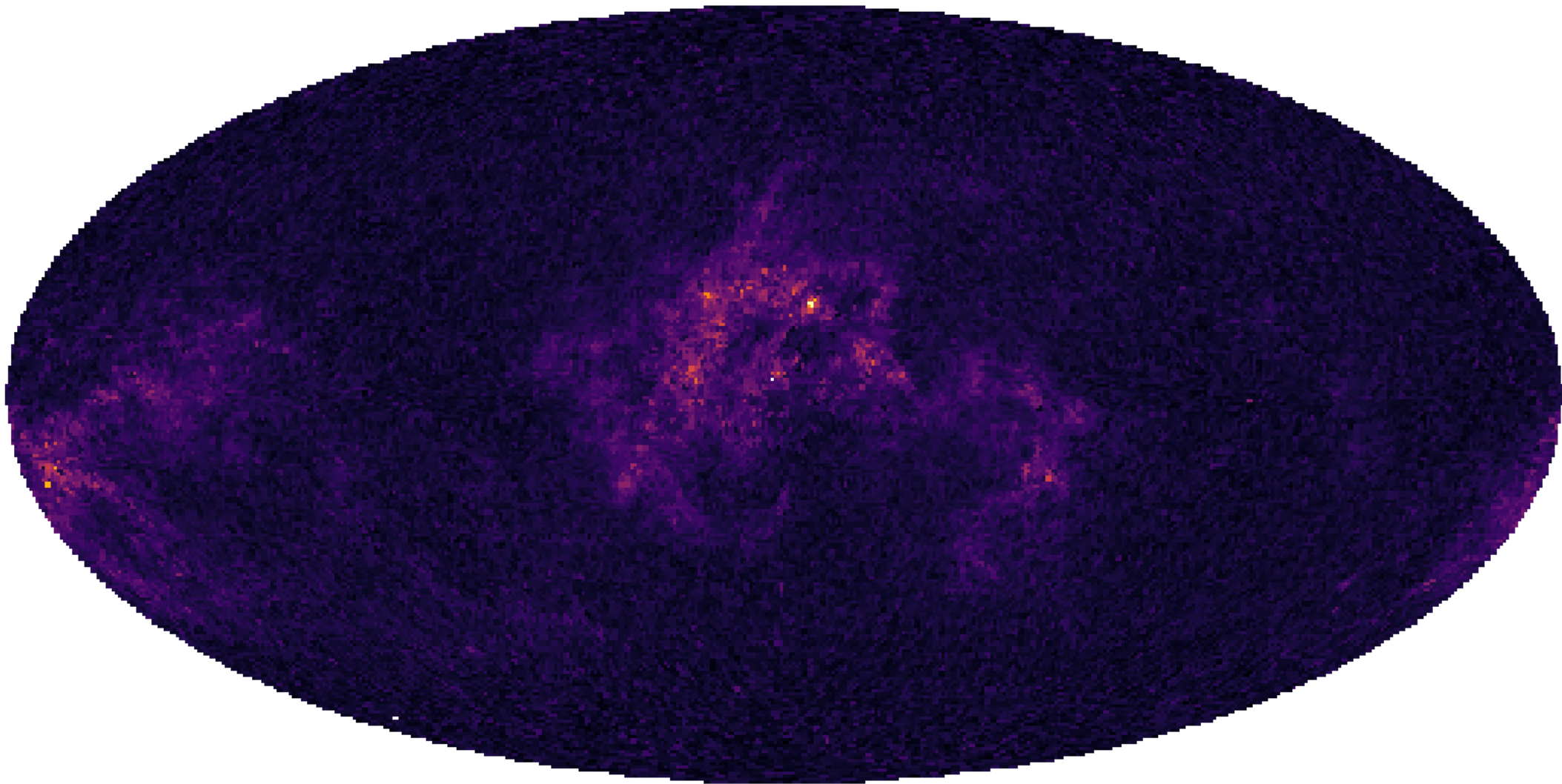
Gaia Dust

Leike & Enßlin (2019)



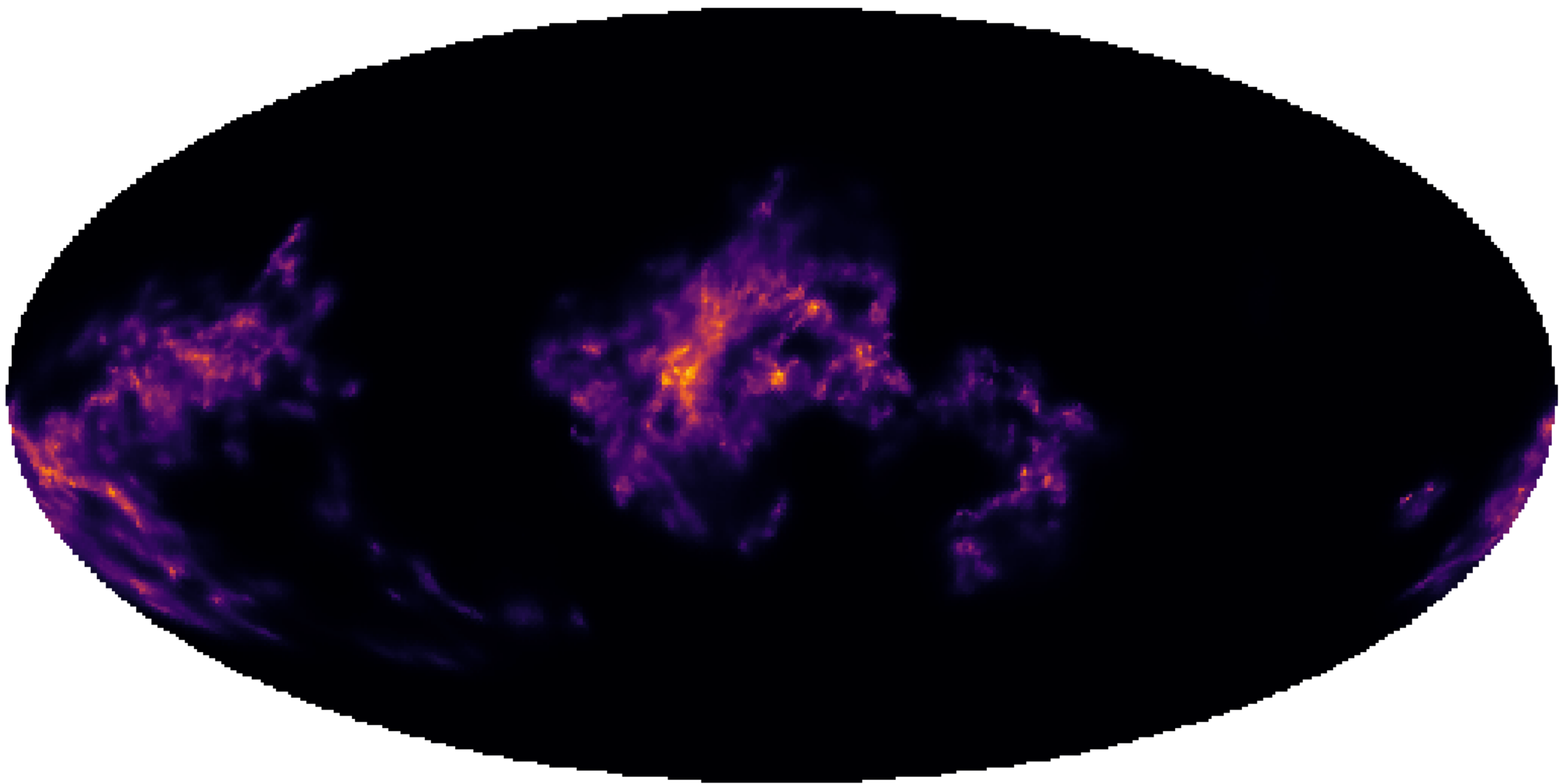
Gaia Dust

Leike & Enßlin (2019)



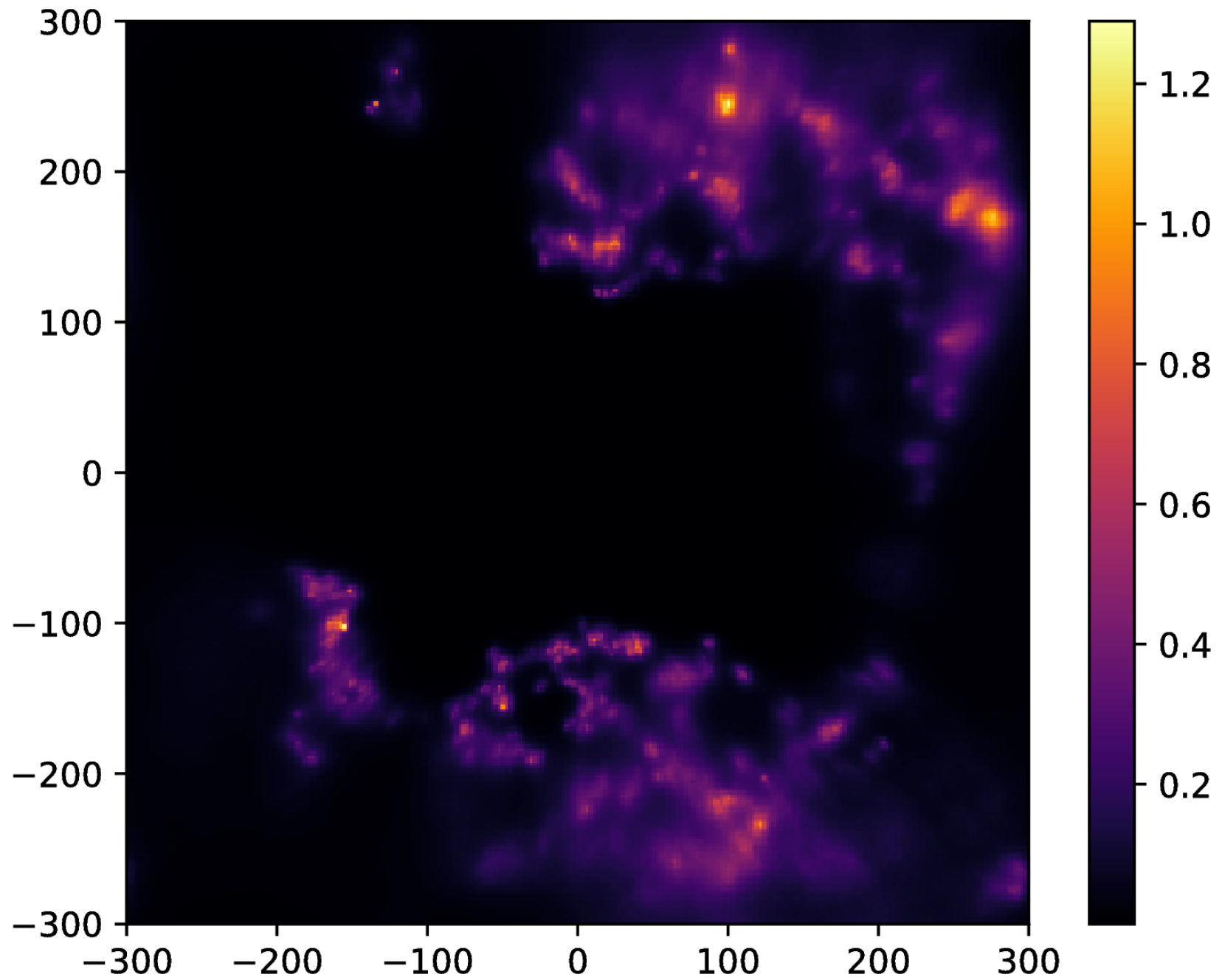
dust density

Leike & Enßlin (2019)



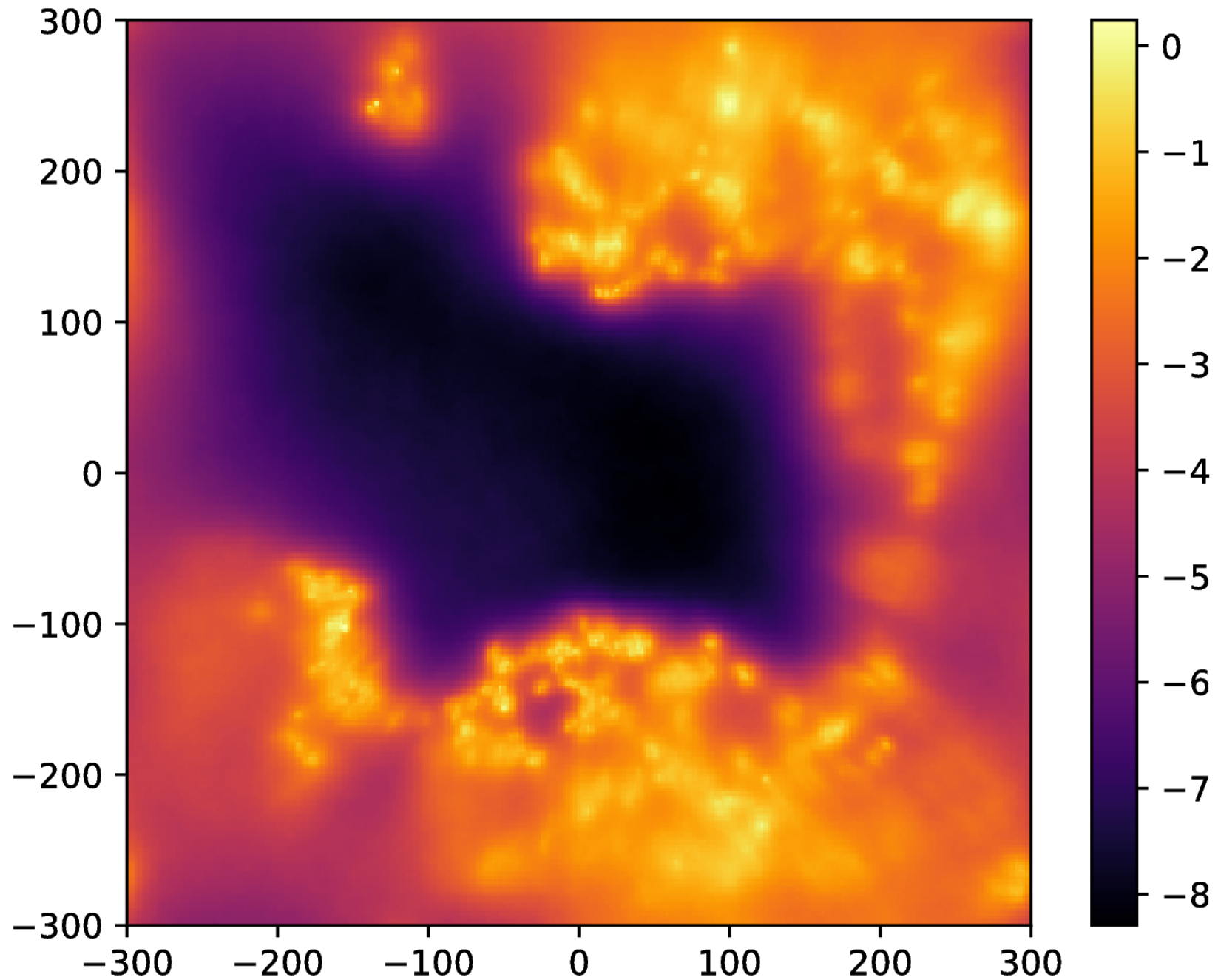
dust density

Leike & Enßlin (2019)



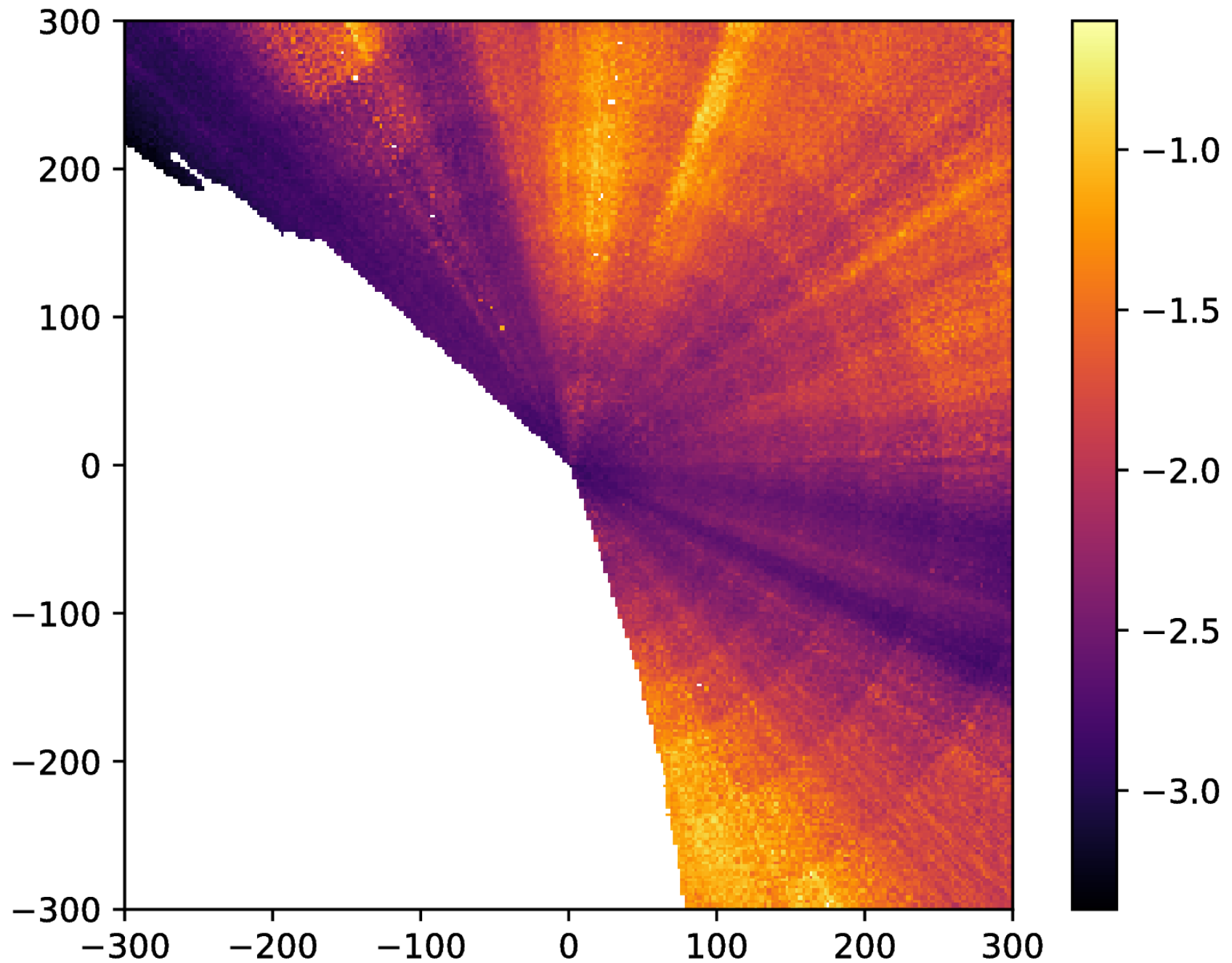
log dust density

Reimar Leike et al. (2019)



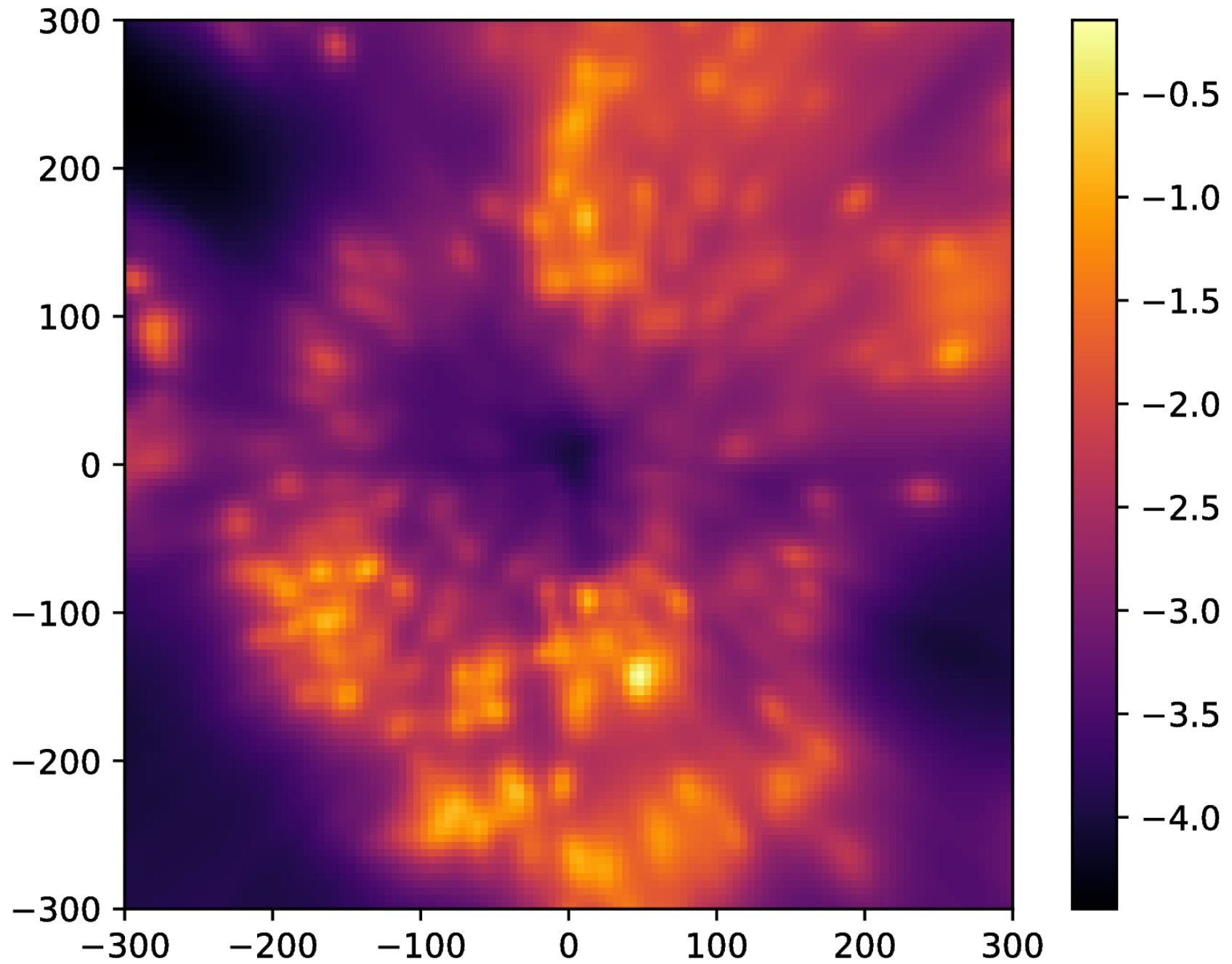
log dust density

Green et al. (2018.)



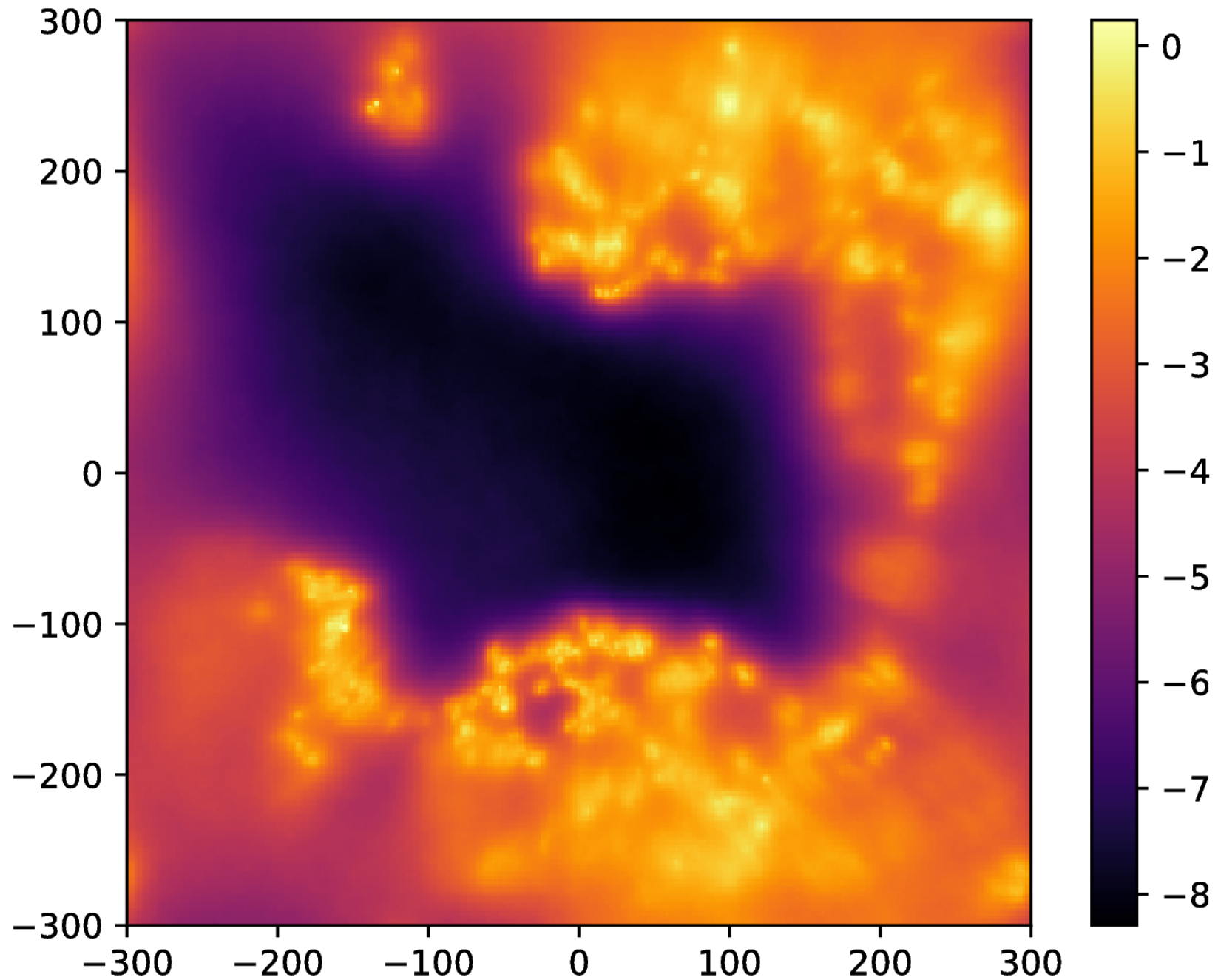
log dust density

Lallement (2018)

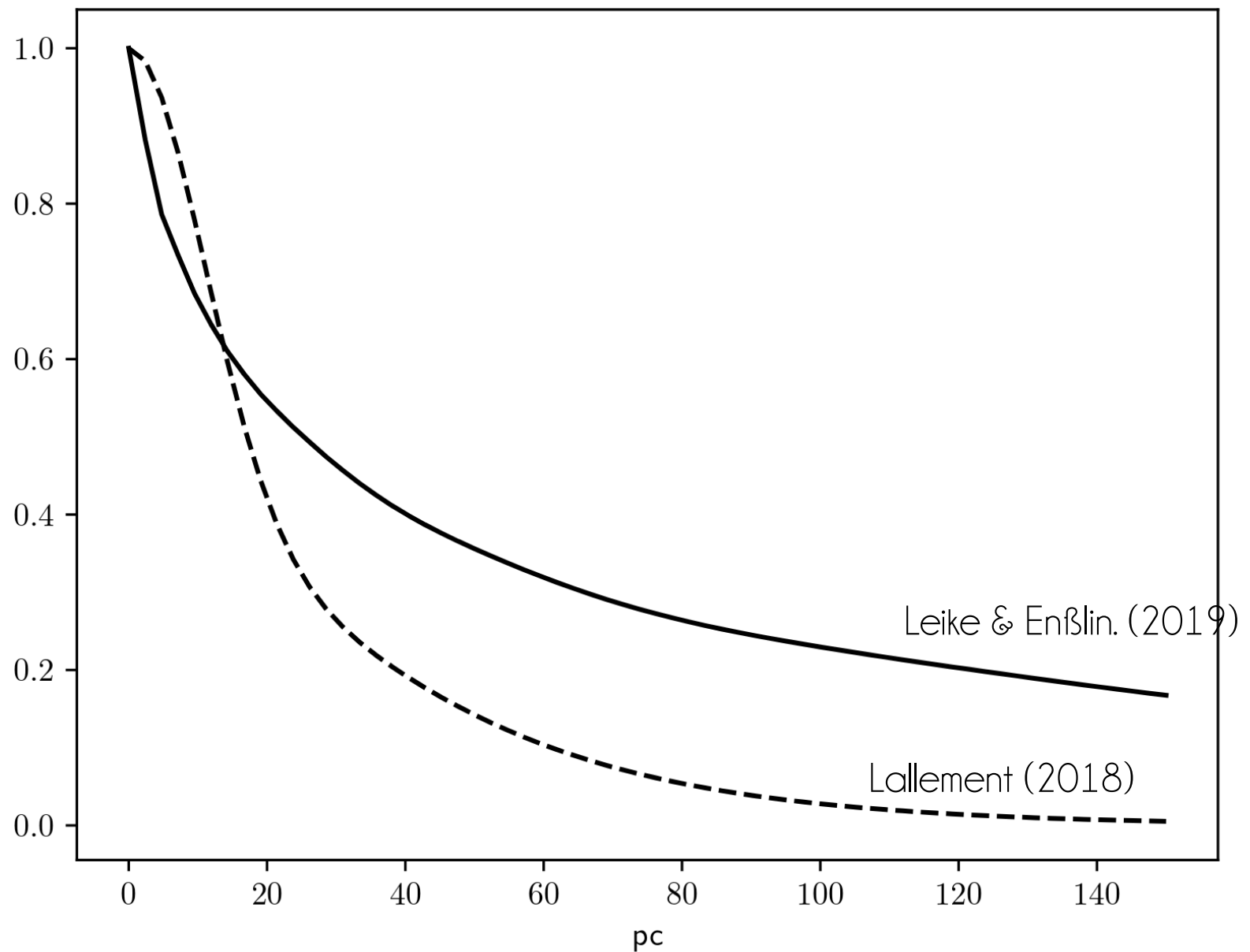


log dust density

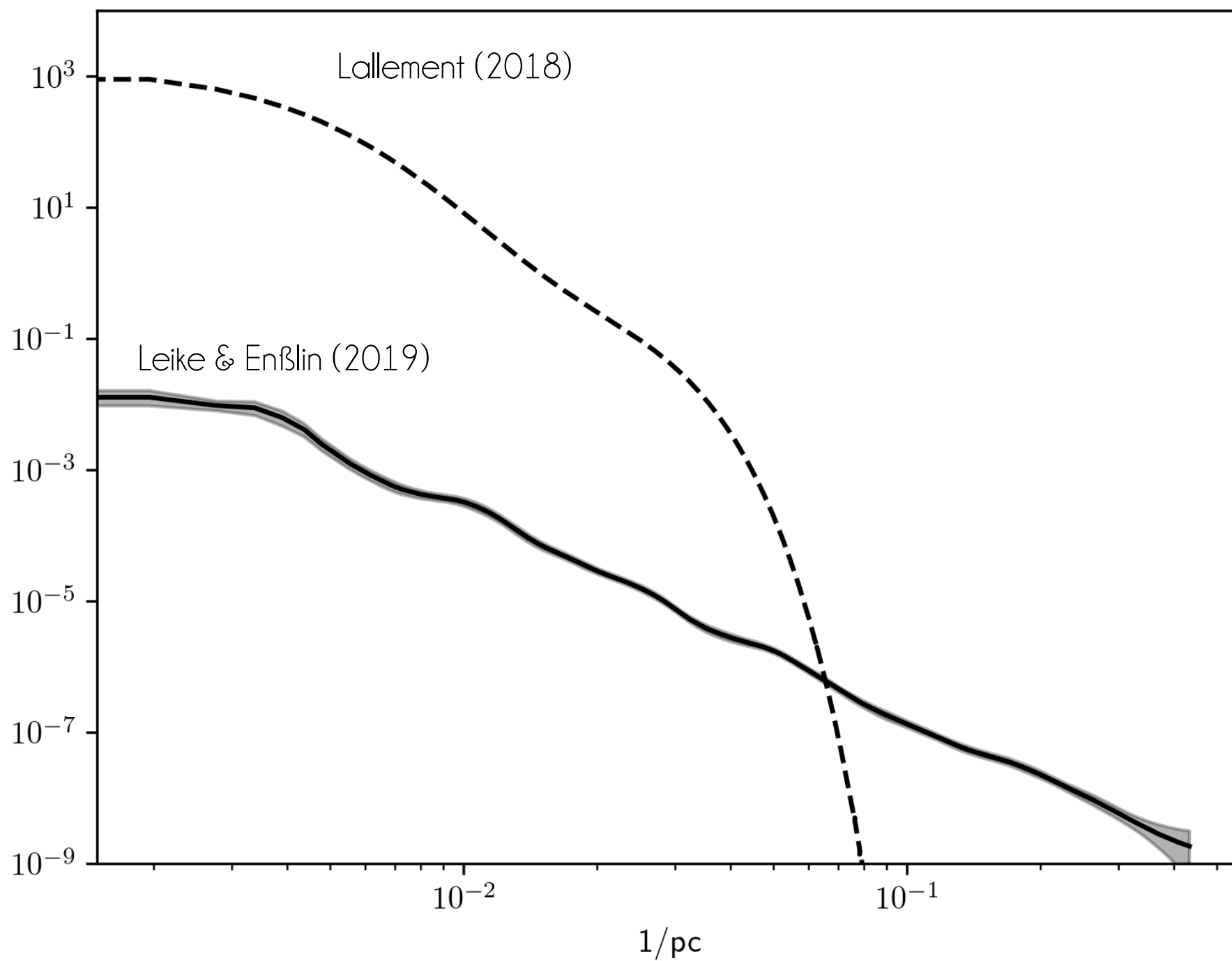
Reimar Leike et al. (2019)



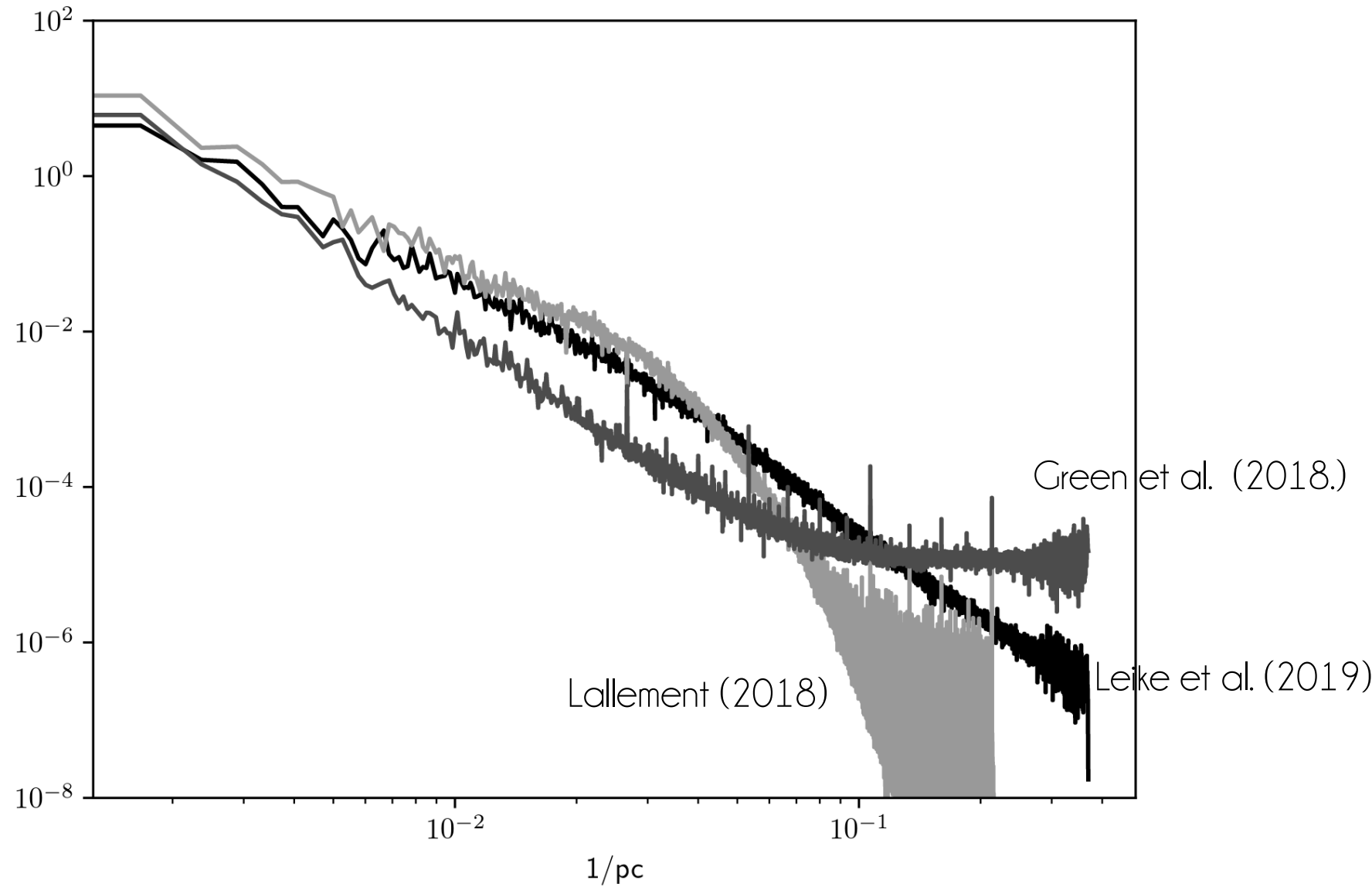
assumed 2-point correlation



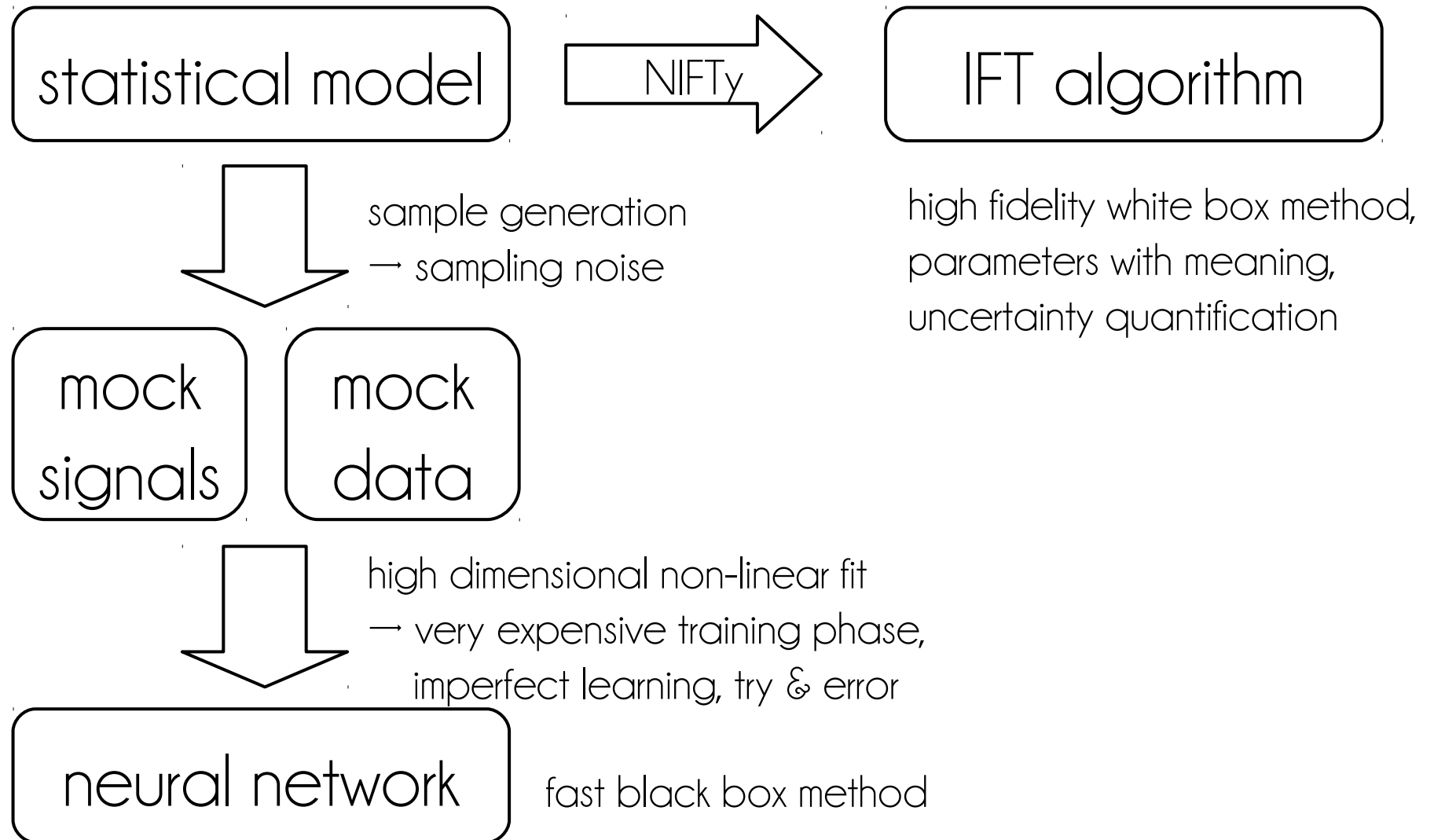
assumed power spectra

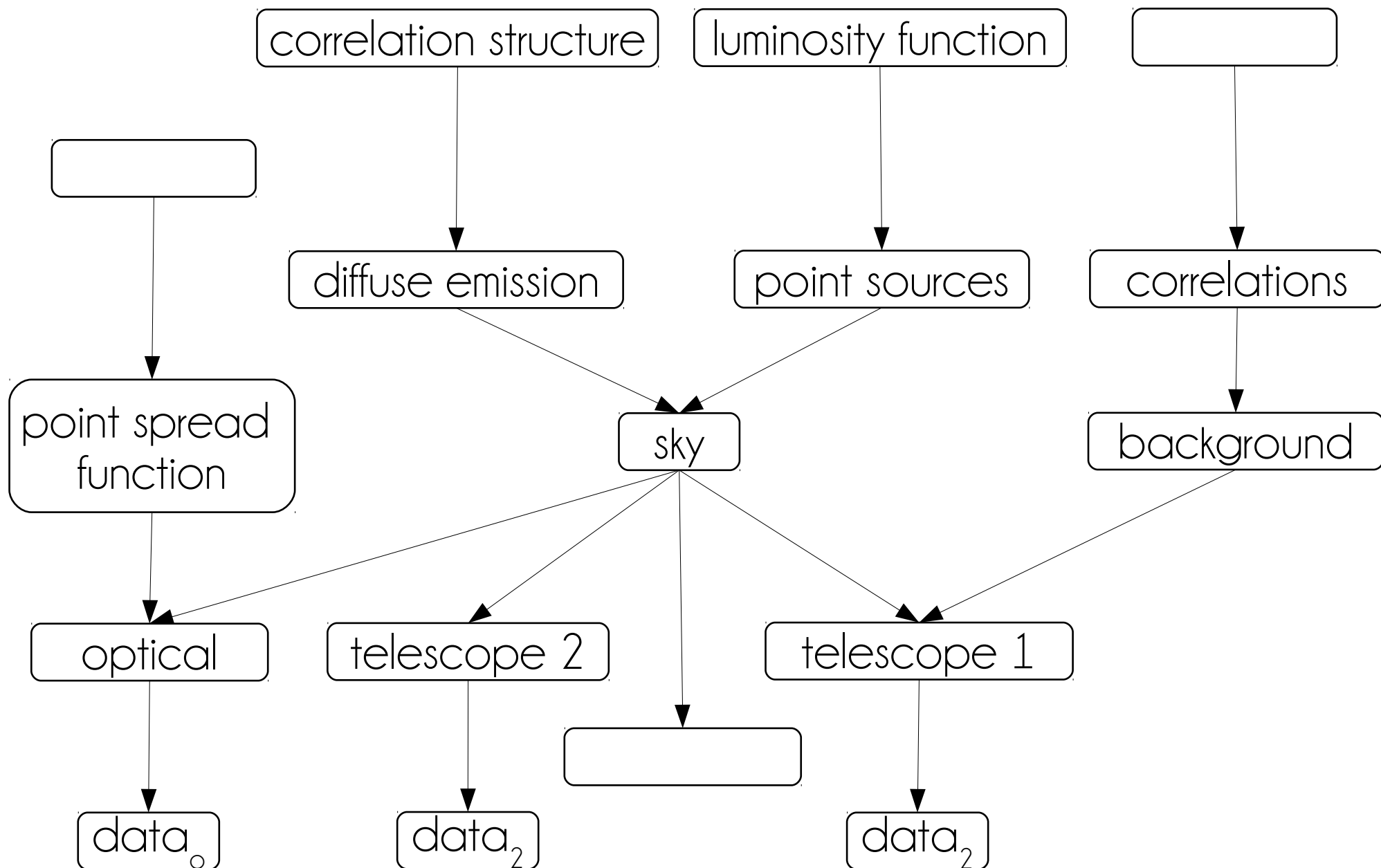


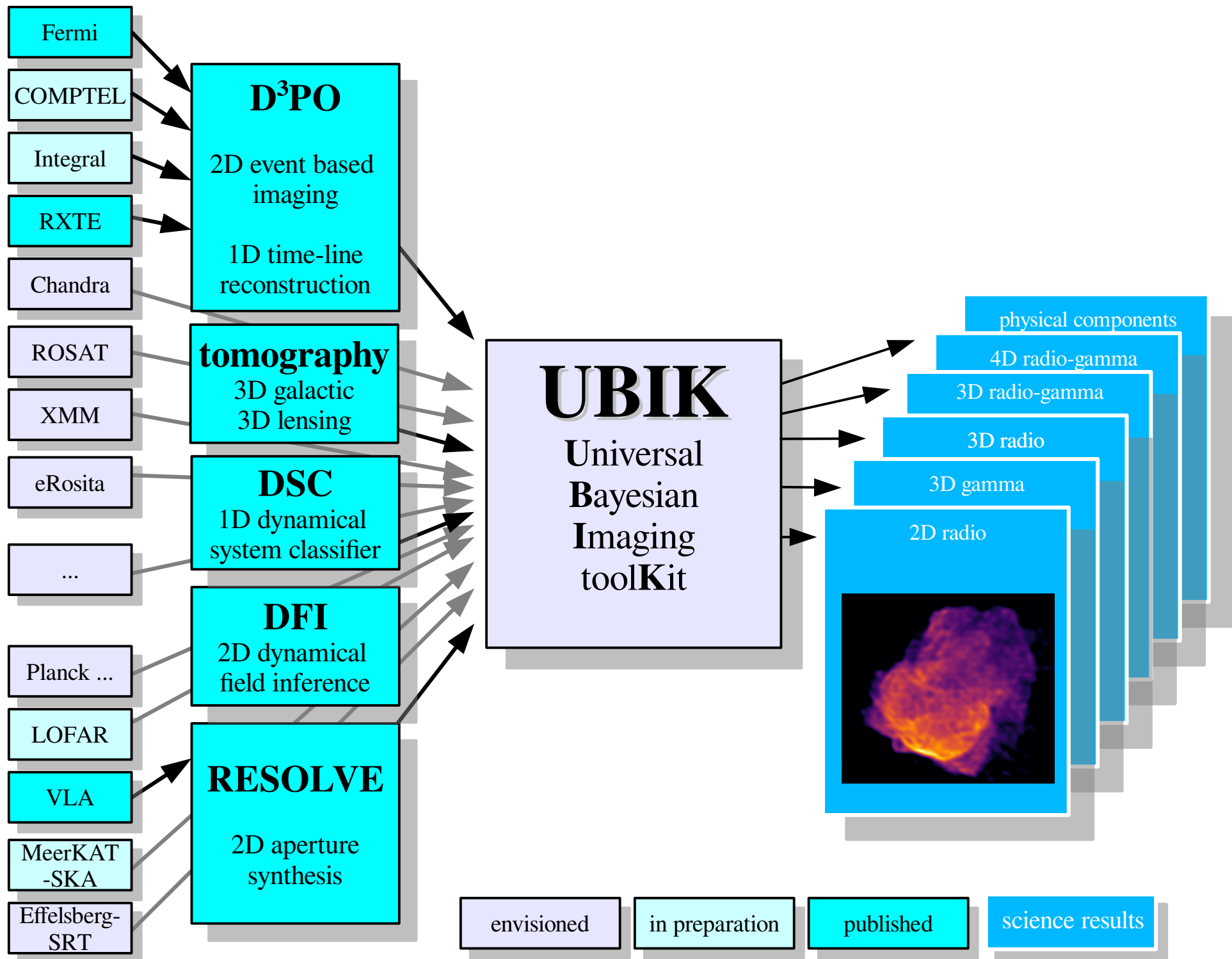
observed power spectra

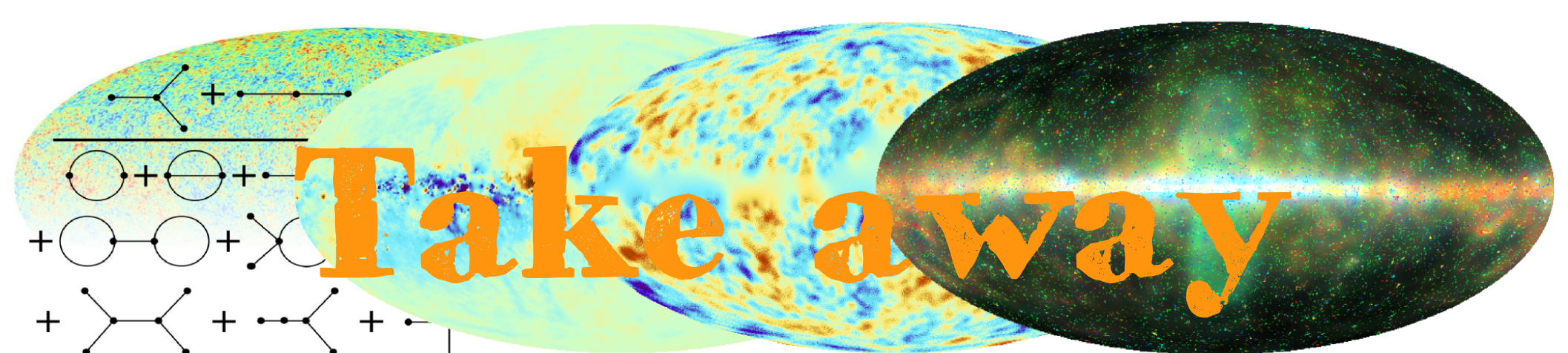


neural networks vs. information field theory









Take away

Imaging goes inference

IFT

- **information field theory**

NIFTy

- **numerical IFT in Python
autodiff/variational inference**

Feb 2019:

UBIK

- **Universal Bayesian Imaging
toolKit - under development**